Backreacted Axions in Type IIA string theory

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Motivations

Super-Planckian field displacements arise in several settings :

- Large Field inflation requires $\Delta\phi \geq M_{\mathsf{Pl}}$.

$$\frac{\Delta\phi}{M_{\mathsf{Pl}}} \geq \mathcal{O}(1)\sqrt{\frac{r}{0.01}} \qquad \text{(Lyth Bound)}$$

  Does not need to be saturated.

- Relaxion : Need large field excursions to naturally cross the critical point of small Higgs mass.

Does Quantum Gravity give constraints on the possible field excursions, in particular can

$$\frac{\Delta\phi}{M_{\mathsf{Pl}}} \geq 1?$$

Framework

We try to answer that question in **type IIA** string theory. At tree-level, we have [Grimm and Louis, 2005] :

$$K = -\ln 8\mathcal{V} - \ln(S + \overline{S}) - 2\ln \mathcal{V}'$$

$$W = e_0 - ih_0 S - ih^i U_i - f_\lambda T^\lambda + \frac{1}{2} k_{\lambda\rho\sigma} \tilde{f}^\lambda T^\rho T^\sigma + \frac{f_0}{6} k_{\lambda\rho\sigma} T^\lambda T^\rho T^\sigma$$

$$\mathcal{V} = \frac{1}{6} k_{\lambda\rho\sigma} t^\lambda t^\rho t^\sigma \qquad \mathcal{V}' = \frac{1}{6} d_{ijk} v^i v^j v^k \qquad u_i = \partial_{v^i} \mathcal{V}'$$

Axions are imaginary parts of the moduli.

$$S = s + i\sigma \qquad U_i = u_i + i\nu_i \qquad T^\lambda = b^\lambda + it^\lambda$$

Only one combination of axions is perturbatively massive. The rest gets massive through non-perturbative effects :

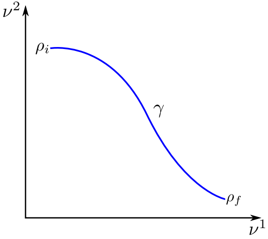
$$W \longrightarrow W + \sum_I e^{-a_I^0 S - a_I^i U_i}$$

Field distances

We are interested in the distance travelled by an axion ϕ in (axion) field space, along a path γ :

$$\Delta\phi = \int_\gamma d\rho \sqrt{g_{ij}\frac{\partial\nu^i}{\partial\rho}\frac{\partial\nu^j}{\partial\rho}}$$

In our setting : $\rho = \sum_i h_i \nu^i$



Before taking the backreaction into effect, the metric does not depend on ρ.

Accounting for the backreaction is done by minimising the other moduli values.

$$\partial_s V = \partial_{T^\lambda} V = \partial_{u^i} V = 0 \qquad \Rightarrow \qquad g_{ij} = g_{ij}(\rho)$$

Field excursions are affected by backreactions.

A Toy Model

What are the consequences of backreaction ?
Consider a simplified toy-model with one modulus of each type :

$$K = -\ln s - 3\ln u - 3\ln t \qquad W = e_0 + ih_0 S - ih_1 U + \frac{im}{6}T^3$$

The massive axion combination is $\rho = e_0 - h_0\sigma + h_1\nu$. The solution to the backreactions equations are [Palti, 2015] :

$$s = \frac{\alpha}{h_0}\rho \qquad u = -\frac{3\alpha}{h_1}\rho \qquad t \simeq 2.0\left(\frac{\rho}{m}\right)^{\frac{1}{3}} \qquad b = 0 \qquad \alpha \simeq 0.4$$

The proper field distance is then

$$\Delta\phi = \int_{\rho_i}^{\rho_f} \frac{d\rho}{2}\left((h_0 s)^2 + \frac{1}{3}(h_1 u)^2\right)^{-\frac{1}{2}} \simeq 0.7\ln\left(\frac{\rho_f}{\rho_i}\right)$$

Properties :

1. Flux independent.
2. The field distance is only logarithmic.

A Toy Model

One can understand this result in terms of scale symmetry :

$$(s, u, \rho) \longrightarrow \lambda(s, u, \rho) \qquad T \longrightarrow \lambda^{\frac{1}{3}} T$$
$$K \longrightarrow K + \text{cst} \qquad W \longrightarrow \lambda W$$
$$V \longrightarrow \lambda^{-3} V$$

s, u, ρ have weight one under the symmetry, meaning that

$$s = s(\rho) \longrightarrow \lambda s = s(\lambda \rho) \qquad \Rightarrow \qquad s, u \propto \rho$$

Then the backreacted inverse metric g^{ij} has weight two.

$$\Delta\phi = \int_{\rho_i}^{\rho_f} \frac{d\rho}{\sqrt{h_i g^{ij}(\rho) h_j}} = \beta(h_0, h_1) \ln\left(\frac{\rho_f}{\rho_i}\right)$$

Backreaction equation forces β to be a pure number.

The scale symmetry allows us to analyse the system generally.

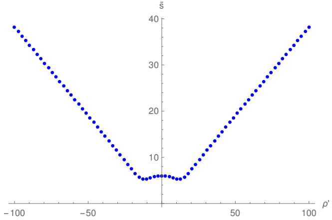
The general 1 modulus case

Is this behaviour generic ?

Consider most general IIA case with one modulus of each case :

$$K = -\ln s - 3\ln u - 3\ln t$$

$$W = e_0 + ih_0 S - ih_1 U + ie_1 T - qT^2 + \frac{im}{6} T^3$$



The flux numbers ruin the scale symmetry, but for axion vevs larger than a quantity we call ρ_{crit}, we recover the toy model :

$$\Delta\phi(\rho) \simeq 0.7 \ln \rho \qquad \text{if } \rho \gg \rho_{\text{crit}} \sim \left| m^3 (e_1 - 2q^2) \right|^{\frac{3}{2}}$$

Flux numbers seem to shield the moduli from vev backreactions.

Flux numbers as order parameters

$$W = e_0 + ih_0 S - ih_1 U + ie_1 T - qT^2 + \frac{im}{6} T^3$$

To "restore" the scale symmetry, we can assign a spurious transformation to the two flux numbers :

$$e_1 \longrightarrow \lambda^{\frac{2}{3}} e_1 \qquad q \longrightarrow \lambda^{\frac{1}{3}} q$$

With some field redefinitions, the potential depends only on one combination of flux numbers :

$$V = h_0 h_1^3 m \widetilde{V}(\tilde{s}, \tilde{u}, \tilde{T}, \rho', f) \qquad f = -m^3(e_1 - 2q^2)$$

So that

$$\rho'_{\text{crit}} \propto f^{\frac{3}{2}}$$

Distance travelled to the critical value

The field distance is again constrained by the scale symmetry ! In particular, the distance travelled until the logarithm behaviour is

$$\Delta\phi_{\text{crit}} = \int_{\rho_i}^{\rho_f} \frac{d\rho}{\sqrt{h_i g^{ij} h_j}} = \int_0^{{\rho'}_{\text{crit}}} \frac{d\rho'}{\sqrt{\delta_{ij}\tilde{g}^{ij}}} = G\left(\frac{{\rho'}_{\text{crit}}}{f^{\frac{3}{2}}}\right) = r$$

Since ${\rho'}_{\text{crit}} \propto f^{\frac{3}{2}}$, r is a pure number.

Consequence

It is impossible to arbitrarily delay the logarithmic behaviour with fluxes !

More precisely, we find

$${\rho'}_{\text{crit}} \simeq 1.7 f^{\frac{3}{2}} \qquad r \simeq 0.9 \text{ (In Planck units)}$$

Generic behaviour

Those properties seem to be generic :

- 2 complex structure moduli : $\mathcal{V}' = \sqrt{u_1}\left(u_2 - \frac{2}{3}u_1\right)$. It can be shown that $\Delta\phi$ does not depend on h_i, critical excursion cannot depend on f because $\mathcal{V}'$ has weight 1.

  Argument seems to generalise to arbitrary number of complex structure moduli (via properties of Kähler potential), but no proof yet.

- Neveu-Schwarz axion b : Same behaviour. $\Delta\phi_{\text{crit}} \simeq 0.6$

- Twisted torus :

$$W = e_0 + ialS - iblU + ie_1T - qT^2 + \frac{im}{6}T^3 + aST - bTU$$

  Two "dimensionful" parameters f and $\tilde{l}$. Complicated system, but can solve for numerical values :

$$-100 \leq f \leq -3 \qquad 1 \leq \tilde{l} \leq 100 \qquad 2 \leq \Delta\phi_{\text{crit}} \leq 3.5 \qquad \rho_{\text{crit}} \sim \left(-\frac{f}{\tilde{l}^2}\right)^{\frac{3}{4}}$$

Conclusion

- Considering backreaction effects in **type IIA** constrains field excursions.

- The field excursion until the critical value is flux independent and sub-Planckian :

$$\Delta\phi_{\text{crit}} \lesssim \mathcal{O}(1)$$

- A spurious symmetry of the potential implies that after a value of the axion v.e.v ρ_{crit}

$$\Delta\phi \sim \ln\rho \qquad \rho > \rho_{\text{crit}}$$