The Weak Gravity Conjecture in different spacetimes

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- Weak Gravity Conjecture: Very useful tool used recently to constrain large field inflation models and relaxation [De La Fuente-Saraswat-Sundrum '14, Rudelius '14,'15, MM-Uranga-Valenzuela '15, Brown-Cottrell-Shiu-Soler '15 (x2), Bachlechner-Long-McAllister '15, Hebecker-Mangat-Rompineve-Witkowski '15, Junghans '15, Heidenreich-Reece-Rudelius '15 (x3), Palti '15, Kooner-Parameswaran-Zavala '15, Ibañez-Montero-Uranga-Valenzuela '15, Hebecker-Rompineve-Westphal '15, Fonseca-de Lima-Machado-Matheus '16, Parameswaran-Tasinato-Zavala '16, Baume-Palti '16, (García-Valdecasas)-Uranga '16].
- Strong forms more constraining [Brown-Cottrell-Shiu-Soler '15, Heidenreich-Reece-Rudelius '15]
- WGC works in every string theory example so far (even perturbative proof for heterotic [(Arkani-Hamed)-Motl-Nicolis-Vafa '06]).
- Heuristic arguments based on BH remnants
- Yet no formal, general proof! Work in progress with **G. Shiu** and **P. Soler** (UW-Madison) along this direction.

The setup

We will look at the WGC in AdS spacetimes. . . [Nakayama-Nomura '15]

. . . and in three dimensions.

Pros & cons:

- Behavior of gravity & gauge fields much simpler.
- Greatly enhanced CFT symmetry group.
- Extra constraints on CFT, such as modular invariance.
- Main con: $d = 3$ so different from $d > 3$ that any relationship to higher d WGC is uncertain at best.

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Gravity in three dimensions

In $d = 3$, gravity is topological, since $R_{\mu\nu\alpha\beta}$ is a function of the metric. In AdS $\exists$ black hole [Bañados, Teitelboim, Zanelli '92]

$$ds^2 = -\left(-8GM + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right) dt^2 + \frac{dr^2}{-8GM + \frac{r^2}{l^2} + \frac{J^2}{4r^2}} + r^2 \left(d\phi - \frac{Jdt}{2r^2}\right)^2$$

with horizon at

$$r_+ = l \left[4GM \left(1 + \sqrt{1 - \left(\frac{J}{Ml}\right)^2}\right)\right]^{\frac{1}{2}}.$$

Notice horizon is of cosmological size!

Gauge fields in 3d

Consider a compact $U(1)$ gauge theory in 3d. Major points:

- In 3d, compact $U(1)$ confines [Polyakov '77] unless we add a Chern-Simons term

$$\frac{\mu}{2}\int F\wedge A,\quad \mu\equiv\frac{N}{2\pi^2}e^2$$

- This modifies e.o.m:

$$d*F=*j_e+\mu F$$

- And Gauss' Law:

$$\int_{S^1}*F=Q_e+\mu\int_{S^1}A$$

  Total charge can be measured by holonomy of A on S^1 at infinity.

- Chern-Simons term actually required from AdS/CFT. [Kraus '07]

Charged BTZ black holes

BTZ black holes can support electric charge in the form of a flat connection

$$Q_e = -\mu^2 \int_{S^1} A$$

This is the 3d analog of the black hole with B-field hair.

[Bowick-Giddings-Harvey-Horowitz-Strominger '88]

- No backreaction on metric, even w. higher derivative corrections. Contrast with $d > 3$. Related to scalar no-hair.
- No apparent extremality bound for Q.
- WGC talks about *superextremal* particles.

The CFT perspective

Weakly coupled AdS_3 is dual to CFT_2 at large central charge

$$c = \frac{3l}{2G}.$$

Bulk $U(1)$ is dual to CFT current $j(z)$ at level N:

$$[j_m, j_p] = N\delta_{m+n,0}, \quad [L_m, j_p] = -p j_{p+m}.$$

- j_0 is proportional to Q, bulk electric charge.
- $[L_0, Q] = [\tilde{L}_0, Q] = 0$: Electric charge is exactly conserved.
- Universal contribution (Sugawara construction) to L_0:

$$L_0 = L_0' + \frac{Q^2}{2N}$$

BH threshold

Standard lore: Only very high dimension CFT operators can be dual to a BH geometry. We find

$$L_0 \geq \frac{c}{24} + \frac{Q^2}{2N}.$$

This is the black hole threshold.

- It is an extremality-like bound: Charged states below are lighter than any black hole (hence superextremal).
- WGC $\leftrightarrow$ show $\exists$ operators below BH threshold.
- Also required by agreement of CFT result (Cardy formula) and semiclassical entropy computation.

Modular invariance

The CFT partition with chemical potential

$$Z(\tau, z) = \text{Tr}\left(q^{L_0 - \frac{c}{24}} \bar{q}^{\tilde{L}_0 - \frac{\tilde{c}}{24}} e^{2\pi i z Q}\right)$$

satisfies $Z(\tau, z) = Z(\tau, z+1)$ due to charge quantization.
Modular invariance implies

$$Z(\tau', z') = \exp\left(i\pi N \frac{z^2}{c\tau + d}\right) Z(\tau, z), \quad \tau \to \frac{a\tau + b}{c\tau + d}, \quad z \to \frac{z}{c\tau + d}$$

Together, these mean

$$Z(\tau, \tau) = \exp(i\pi N \tau) Z(\tau, 0).$$

or

$$Z(\tau, 0) = \text{Tr}\left(q^{L_0 - \frac{c}{24} + Q + \frac{N}{2}} \bar{q}^{\tilde{L}_0 - \frac{\tilde{c}}{24}}\right)$$

Modular invariance II

Conclusion: the spectrum is invariant under *spectral flow* by N units

$$L_0 \to L_0 + Q + \frac{N}{2}, \quad Q \to Q + N, \quad \tilde{L_0} \to \tilde{L}_0.$$

Acting on the vacuum, spectral flow produces a state of charge

$$Q = kN \quad \text{and} \quad L_0 = k^2 \frac{N}{2},$$

which are below the BH threshold: These states satisfy the (strong & Lattice) WGC in three dimensions.

Comparison with WGC heuristics

Usual WGC heuristics do not apply here for several reasons:

- No tunable coupling: Fixed by AdS radius & level N.
- Large AdS black holes do not evaporate.
- In 3d, small BTZ black holes (compared to l) receive important quantum corrections.

Electric charge behaves like global charge; even this seems OK because of the last point.

The $\mathbb{Z}_N$ charge

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Mod. invariance only gives lights states with $Q = kN$: Remnant $\mathbb{Z}_N$ charge does not have a WGC.

- BH charge observable via Aharonov-Bohm kind of experiments (BH with discrete electric hair [Coleman, Preskill, Wilczek '92])
- Some constraints on spectrum from modular invariance (modular bootstrap [Benjamin, Dyer, Fitzpatrick, Kachru '16]), but
- Not enough to establish WGC in 3d (explicit counterexample).

No reason (stringy or remnants-based) to expect WGC for discrete symmetries anyway, even in $d > 3$.

Summary

- Drastic differences for charged BH's in 3d.
- Modular invariance + compact gauge group = Strong/Lattice WGC in 3d.
- Black hole heuristics not relevant/applicable
- Remnant $\mathbb{Z}_N$ charge does not have WGC from modular invariance alone

Outlook:

- How much can we take to $d > 3$?
- Any heuristic motivation for 3d WGC?

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Thank you very much!