Gravitational Instantons, Moduli Stabilisation and Axion Inflation

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Outline

Questions and issues to be addressed:

1. Gravitational Instantons and Axion Inflation!? (Motivation part)
2. Types of Gravitational Instantons: Wormholes, Extremal and Cored Instantons
3. Moduli Dynamics: Can Gravitational Instantons survive?
4. Can Gravitational Instantons finally constrain Axion Inflation?
5. If time permits: Comments related to Weak Gravity Conjecture (WGC)...

Motivation: Gravitational Instantons and Axion Inflation!?

- **Gravitational Instantons**: Finite-action solutions to the eqs. of motion of Euclidean gravity theory.
- Consider **shift-symmetric axion θ coupled to gravity**:

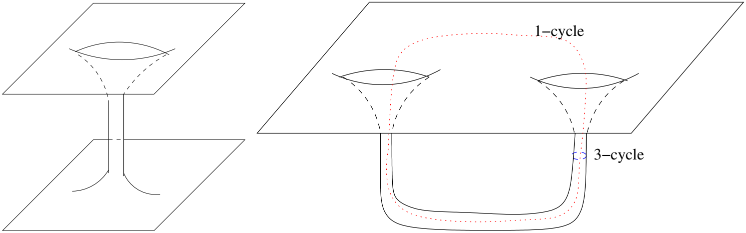
$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}R + \frac{1}{2}\mathcal{F}H_{\mu\nu\rho}H^{\mu\nu\rho}\right] + S_{\rm boundary} \ ,$$

  where $H = dB$, B 2-form gauge potential, dual to θ via $H = f_{\rm ax}^2 \star d\theta$, and $\mathcal{F} = 1/(3!f_{\rm ax}^2)$ (constant for the moment), $f_{\rm ax}$ axion-decay constant.
- N.B.: Description in terms of B more convenient (θ-description involves complex saddles in path-integral).
- Gravitational Instanton solutions to this gravity-axion system: **Giddings-Strominger wormholes**!

[Giddings & Strominger (1988)]

[Abbott & Wise ('89), Coleman & Lee ('89), Kallosh et al. ('95), ...]

Motivation: Gravitational Instantons and Axion Inflation!?



(a) Connection to other universe

(b) Connection to same universe

- Metric (input: rotational symmetry):

$$ds^2 = \left(1 + C/r^4\right)^{-1} dr^2 + r^2 d\Omega_3^2, \qquad C < 0,$$

  wormhole radius $r_0 \equiv |C|^{1/4}$.
- H-flux is quantised as

$$\int_{S^3} H = n \in \mathbb{Z}.$$

Motivation: Gravitational Instantons and Axion Inflation!?

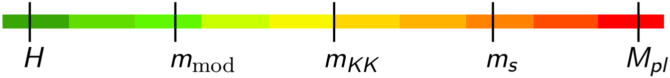
- Gravitational Instanton contributions $\delta V \sim \cos(n\theta)e^{-S}$.

  $\hookrightarrow$ Lukas' talk tomorrow, [Hebecker, PM, Theisen, Witkowski. - to appear]

- Those **instanton corrections** may be enough to **constrain some axion inflation models**... [Montero et al. (2015)]
  No constraints for inflation with many axions $N \gg 1$.

  [Bachlechner et al. (2015)]

- But ... in our Einstein-axion system heavy fields are integrated out. **Breakdown at compactification scale?**



- If yes, then there are no constraints for inflation!
  Argument: [Hebecker, PM, Rompineve, Witkowski (2015)]
  Moduli stabilisation implies $H < m_{\mathrm{mod}}$.
  Trust gravitational instantons only up to cutoff $\Lambda < m_{\mathrm{mod}}$.
  $\Rightarrow$ Strongest effects from gravitational instantons at $H \sim \Lambda \sim m_{\mathrm{mod}}$.

Motivation: Gravitational Instantons and Axion Inflation!?

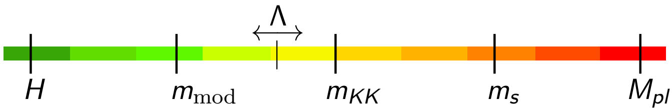
- Instanton action $S \sim n/f_{ax}$, maximal curvature $R \sim f_{ax}/n$.
  Cutoff $\Lambda \sim H$ limits Ricci scalar $R \lesssim \Lambda^2$. (Alternatively, wormhole radius $r_0 \gtrsim \Lambda^{-1}$.)
  Comparison with the energy scale of inflaton:

$$e^{-S} \sim e^{-1/R} \sim e^{-1/\Lambda^2} \sim e^{-1/H^2} \ll H^2$$

$\Rightarrow$ Relevant contributions can only arise **beyond the compactification scale**!

[Hebecker, PM, Rompineve, Witkwoski (2015)]

$\Rightarrow$ Thus, we need to take into account the **presence of moduli fields** and push Λ beyond m_{mod}:



Gravitational Instantons with a *massless* scalar field

- **First step:** include a **massless scalar** φ which **couples to** θ:

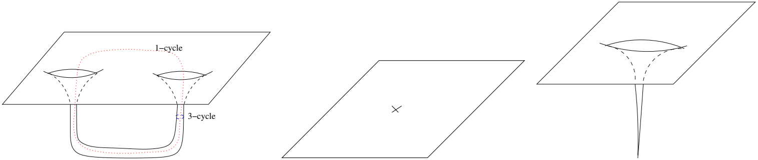
$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}R + \frac{1}{2}\mathcal{F}(\varphi)H^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right],$$

where $\mathcal{F}(\varphi)$ accounts for the coupling of φ to θ.

- Metric is again

$$ds^2 = \left(1 + C/r^4\right)^{-1} dr^2 + r^2 d\Omega_3^2,$$

but now distinguish between $C < 0$, $C = 0$ and $C > 0$:



- $C < 0$ wormhole instantons, $C = 0$ extremal instantons, $C > 0$ cored instantons.

[Heidenreich et al. (2015)]

Gravitational Instantons with a *light* scalar field

- **Second step:** Let φ be the lightest modulus $\rightarrow$ **add mass term** with $m \ll 1$:
$$S = \int d^4x \sqrt{g} \left[-\frac{1}{2}R + \frac{1}{2}\mathcal{F}(\varphi)H^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}m^2\varphi^2 \right].$$
- Approximating $V = 1/2m^2\varphi^2$ only valid for small displacements $\varphi < \varphi_{\text{max}} = \mathcal{O}(1)$. (Convention $\varphi \rightarrow 0$ as $r \rightarrow \infty$.)
- Metric looks like $ds^2 = \lambda(r)dr^2 + r^2 d\Omega_3^2$, where there is in general no analytical expression for $\lambda(r)$.
- **However:** For $r < \mathcal{O}(1)/m$ we **can neglect the mass term** and φ is approx. solution of massless scalar!
- Thus, for $r < \mathcal{O}(1)/m$ can classify the grav. inst. by parameter C.
- For $r > \mathcal{O}(1)/m$ can show that φ is small and goes to zero as $r \rightarrow \infty$.

$\Rightarrow$ Can concentrate on dynamics for $r < \mathcal{O}(1)/m$ pretending that φ is massless.

Gravitational Instantons with a dilaton

- **Question:** For which values of r is $\varphi(r) = \mathcal{O}(1)$?
- **Case study** necessary: How does $\mathcal{F}(\varphi)$ look like in the simplest string compactifications?
  - Easy to obtain $\mathcal{F}(\varphi) = e^{-\alpha\varphi}/(3! f_{ax}^2)$.
  - Such **dilatonic couplings** have been studied in previous literature. e.g. [Giddings & Strominger ('88), Bergshoeff et al. ('04), Heidenreich et al. ('15)]
  - We find: α not arbitrary! Few possibilities, e.g. $\alpha = 2\sqrt{2/3}$ (e.g. LVS, large complex structure limit), $\alpha = 2\sqrt{2}$ (Axio-Dilaton), $\alpha = \sqrt{2}$ (large compl. str. in F-Theory).
- N.B.: Wormholes only for $0 \leq \alpha < 2\sqrt{2/3}$ (GS-wormhole for $\alpha = 0$).
- Wormholes ($C < 0$): φ is maximal at $r = r_0$ (wormhole radius). $\varphi(r_0)$ independent of n/f_{ax} but $\varphi(r_0) < \mathcal{O}(1)$ for most values of α!
- Extremal ($C = 0$) or cored instantons ($C > 0$): $\varphi \to \infty$ as $r \to 0$. $\varphi(r) > 1$ can be "delayed" by making n/f_{ax} sufficiently small.

Lightest Gravitational Instantons (Gravity+Axion+Dilaton)

- **Moreover: We trust the system only down to r_c $(\sim \Lambda_{KK}^{-1})$.**
- Wormholes: Impose $r_0 \gtrsim r_c$. Can show that

$$S \gtrsim (2\pi^2)\sqrt{6}\frac{2}{\alpha}\tan\left(\frac{\alpha\pi}{4}\sqrt{\frac{3}{2}}\right) r_c^2 \geq \underbrace{3\pi^3 r_c^2}_{\text{GS-wormhole}} \quad .$$

- Extremal instantons: $r \gtrsim r_c$, ensure that contribution ΔS from $(0, r_c)$ is negligible (compared to S). Can show $\varphi(r_c)$ under control. Result:

$$S > \frac{16\pi^2}{\alpha^2} r_c^2 .$$

- Roughly: S^3 should only as big as the self-dual volume of the extra-dimensions. Details differ by π-factors.
E.g.: $r_c = \sqrt{4\pi}(2\pi^2)^{-1/3}$ vs. $r_c = 1/\sqrt{\pi}$.
- $e^{-S} \ll 10^{-8} \sim V_{\text{infl}}$ for wormholes, $e^{-S} > 10^{-8}$ for extremal instantons possible, but e^{-S} very sensitive to $\mathcal{O}(1)$ factors!

Further comments and conclusions

see [Hebecker, PM, Theisen, Witkowski (to appear)]

Further comments

- Cored gravitational instantons may be unstable and hence not relevant for inflaton. (In 5d they correspond to non-extremal RN black holes which can decay due to WGC.)
- Charge-to-mass ratio of wormhole instantons bigger than of extremal instantons. Are wormholes the objects satisfying a WGC for gravitational instantons?

Conclusions

- Sufficient control over the dynamics of moduli coupled to the axion.
- Breakdown of effective description of gravitational instantons expected at KK-scale. Unfortunately r_c hard to determine.
- Wormhole contributions sufficiently suppressed. Not so clear for extremal instantons (Large α possible? $\mathcal{O}(1)$-factors?).

Thank you for your attention!