Non-perturbative type IIB vacua beyond F-theory

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Work done in collaboration with

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arXiv:1411.4785, arXiv:1411.4786 and work in progress.

General philosophy:

- non-geometric compactifications
- locally geometric solutions are glued together by U-dulity transformations
- here: type IIB solutions
  [Martucci, Morales, Pacifici, 2012] [Braun, Fucito, Morales, 2013]
- non-geometric heterotic compactifications
  [Malmendier, Morrison, 2014] [Gu, Jockers, 2014]
  [Font, Garcia-Etxebarria, Lüst, Massai, Mayrhofer, 2016]

Original inspiration for F-theory:

- 12d auxiliary construction used for type IIB string theory compactifications with axiodilaton varying over the compactification space.
- $SL(2,\mathbb{Z})$ symmetry of 10d type IIB $\leftrightarrow$ geometric $SL(2,\mathbb{Z})$ action on the complex structure of T^2
- $\tau = C_0 + i\, e^{-\phi}$ is mapped to the complex structure parameter of the elliptic fiber
- D7-branes are magnetically charged under the axiodilaton $\rightarrow$ monodromy:

$$\tau \rightarrow \tau + 1$$

Type IIB vacua with:

- $B_2 = C_2 = C_4 = 0$.
- z: complex coordinate on the $9 - 10$ direction
- $\tau(z, \bar{z})$ and $g_{z\bar{z}}(z, \bar{z})$
- solutions which preserve half of the supersymmetries $\longrightarrow$

  τ is holomorphic
- example: 7-brane at $z = 0$

$$\tau(z) \sim \frac{1}{2\pi i} \log(z)$$

- at infinity: spacetime locally flat, deficit angle $\pi/6$

Inspiration for G-theory:

- in lower dimensions: larger U-duality groups
- the $SL(2,\mathbb{Z})$ symmetry of 10d type IIB is replaced by the U-duality group $SO(2,n,\mathbb{Z})$ and interpreted as the action on the space of complex structures of K3.
- the structure of singularities is richer, allowing for monodromies associated to brane charges of various types
- tadpole cancelation: automatically solved by holomorphicity.
- advantage: while the fluxes are complicated multi-valued functions over the base, the geometric description allows for a global description without branch cuts.

- local type IIB solutions on $\mathbb{R}^{1,3} \times T^4 \times \mathbb{C}$
- the metric of the torus g_{mn}, the dilation ϕ, the NS-NS B-field and R-R C_p-fields are assumed to vary over $\mathbb{C}$. All the non-trivial fluxes are assumed to be oriented along T^4.

$$ds^2 = e^{2A} \sum_{\mu=0}^{3} dx_\mu dx^\mu + \sum_{m,n=1}^{4} g_{mn}\, dy^m\, dy^n + 2\, e^{2D}\, |h(z)|^2\, dz\, d\bar{z}$$

$$B = \frac{1}{2}\, b_{mn}\, dy^m \wedge dy^n\ , \quad C_2 = \frac{1}{2}\, c_{mn}\, dy^m \wedge dy^n\ ,$$

$$C_4 = c_4\, dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4\ ,$$

with A, D, $g_{mn}, b_{mn}, c_{mn}, c_4, C_0$ and ϕ varying only over the complex plane.

The scalar manifold of the maximal supergravity in 6d:

$$\mathcal{M}_{\text{IIB on } T^4} = SO(5,5,\mathbb{Z})\backslash\frac{SO(5,5,\mathbb{R})}{SO(5,\mathbb{R})\times SO(5,\mathbb{R})}$$

Consistent truncation of the moduli space to:

$$\mathcal{M} = SO(2,n,\mathbb{Z})\backslash\frac{SO(2,n,\mathbb{R})}{SO(2,\mathbb{R})\times SO(n,\mathbb{R})} \subset \mathcal{M}_{\text{IIB on } T^4}$$

for $n \leq 5$. Remarkably, $\mathcal{M} \simeq \mathcal{M}_{\mathrm{K3,n}}$.

U-duality group $SO(2,n,\mathbb{Z})$ interpreted as the action on the space of complex structures of a K3.

The two-parameter case:

$$SO(2,2,\mathbb{Z}) \simeq SL(2,\mathbb{Z})_\tau \times SL(2,\mathbb{Z})_\sigma \rtimes (\mathbb{Z}_2^{\tau\leftrightarrow\sigma} \times \mathbb{Z}_2)$$

- this is also the T-duality group of the heterotic string on T^2 with no Wilson lines.

The three-parameter case:

$$SO(2,3,\mathbb{Z}) \supset O^+(L^{2,3}) \longrightarrow Sp(4,\mathbb{Z})$$

- this is also the T-duality group of the heterotic string on T^2 with a single Wilson line breaking $E_8 \times E_8$ to $E_7 \times E_8$.

Local solutions, in terms of holomorphic functions $\tau(z)$ and $\sigma(z)$:

$$g_{mn} = e^{\phi - 2A}\,\delta_{mn} \qquad e^{2D} = e^{-2A} = \sqrt{\sigma_2 \tau_2} \qquad e^{-\phi} = \tau_2$$

The non-trivial fluxes are:

$$C_0 = \tau_1 \qquad C_4 = -\sigma_1\, dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4$$

This solution describes general systems of D3- and D7-branes and their U-dual. By T-duality and S-duality one can relate to solutions with intersecting NS5-branes and solutions with intersecting D5-branes.

So we have:

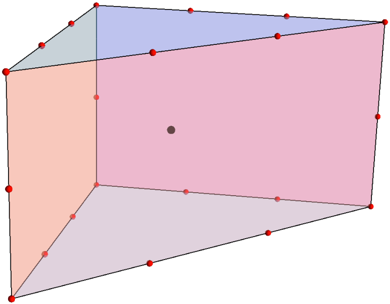
$$\tau = C_0 + i\, e^{-\phi} \qquad \text{and} \qquad \sigma = c_4 + i\,\mathrm{Vol}(T^4)$$

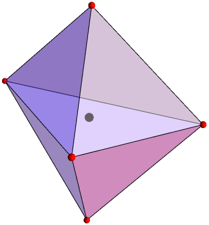
A monodromy:

$$\tau \longrightarrow \tau + q_1 \qquad \sigma \longrightarrow \sigma + q_2 \qquad \text{with} \qquad q_1, q_2 \in \mathbb{Z}$$

around the point z_0, indicates the presence of q_1 D7-branes and q_2 D3-branes at the point z_0. Notation: a brane of charge (q_1, q_2) is located at z_0.

A 2-parameter example:





$$f_{\rm hom} = \sum_{a=0}^{5} c_a \prod_{i=1}^{6} z_i^{\langle w_i^{K3}, v^a \rangle + 1}$$

$$= -\, c_0\, z_1\, z_2\, z_3\, z_4\, z_5\, z_6 + c_1\, z_1^3\, z_2^3 + c_2\, z_3^3\, z_4^3 + c_3\, z_5^3\, z_6^3 + c_4\, z_2^2\, z_4^2\, z_6^2 + c_5\, z_1^2\, z_3^2\, z_5^2\ .$$

Two free parameters:

$$\frac{\xi}{27} = \frac{c_1 c_2 c_3}{c_0^3} \qquad \text{and} \qquad \frac{\eta}{4} = \frac{c_4 c_5}{c_0^2}\ .$$

The complex structure moduli space:

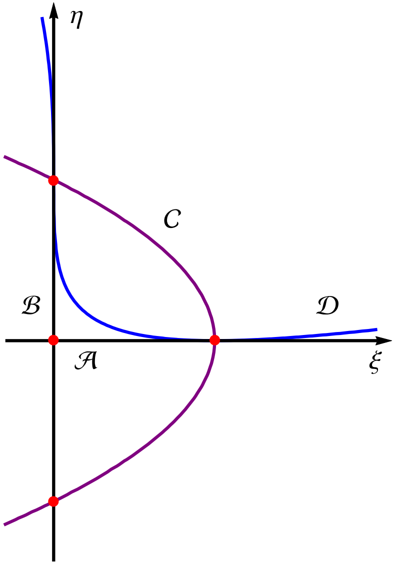
- parametrisation in terms of the periods:

$$\varpi_i = \int_{e^i} \Omega \ .$$

- compute the fundamental period ϖ_0 by direct integration in the large complex structure limit $\xi, \eta \to 0$.
- infer the Picard-Fuchs equations satisfied by ϖ_0,

$$\mathcal{L}_1 \varpi_0 = \mathcal{L}_2 \varpi_0 = 0 \ .$$

- construct the other three periods $\varpi_1, \varpi_2, \varpi_3$ using the method of Frobenius.



$$\mathcal{C}: \; \xi + \eta^2 - 1 \; = \; 0 \; , \quad \mathcal{D}: \; (\xi - 3\eta - 1)^2 - \eta(\eta + 3)^2 \; = 0 \; .$$

Identify τ and σ with the period ratios:

$$\tau^{(1)} = \frac{\varpi_1}{\varpi_0} = \frac{1}{2\pi i}\log\left(\frac{\xi}{27}\right) + \frac{h_{10}}{h_{00}}$$

$$\tau^{(2)} = \frac{\varpi_2}{\varpi_0} = \frac{1}{2\pi i}\log\left(\frac{\xi\,\eta^3}{1728}\right) + \frac{3\,h_{01} + h_{10}}{h_{00}} \; .$$

Thus, by going around the $\mathcal{A}$-divisor, corresponding to $\eta = 0$:

$$\tau^{(1)} \to \tau^{(1)} \quad , \qquad \tau^{(2)} \to \tau^{(2)} + 3$$

By going around the $\mathcal{B}$-divisor, corresponding to $\xi = 0$:

$$\tau^{(1)} \to \tau^{(1)} + 1 \quad , \qquad \tau^{(2)} \to \tau^{(2)} + 1$$

$$j_1 = \frac{6^3}{\xi\eta^3}\left(1 - \xi - \eta - \sqrt{D}\right)^3 , \quad j_2 = \frac{6^3}{\xi\eta^3}\left(1 - \xi - \eta + \sqrt{D}\right)^3 .$$

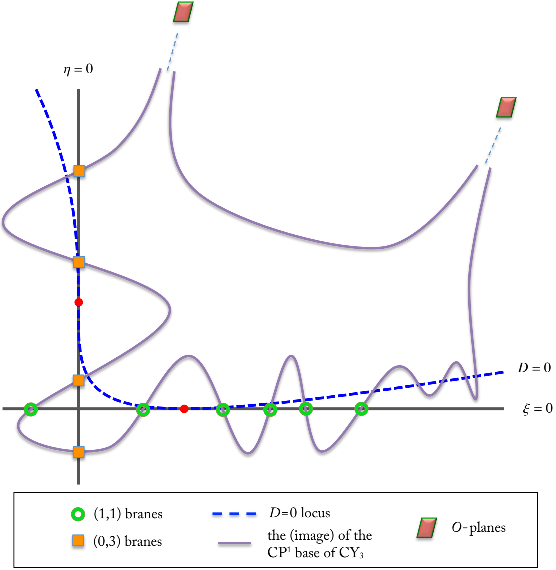


Figure: The embedding of the (real part of the) $\mathbb{CP}^1$ base (in purple) of the Calabi-Yau threefold into the CS moduli space of the $K3$ fiber.

- K3 fibration over $\mathbb{P}^1$:

$$f_{\mathrm{CY}_3,\,\mathrm{hom}} = -\, c_0(z)\, z_1\, z_2\, z_3\, z_4\, z_5\, z_6 + c_1(z)\, z_1^3\, z_2^3 + c_2(z)\, z_3^3\, z_4^3 + c_3(z)\, z_5^3\, z_6^3 + c_4(z)\, z_2^2\, z_4^2\, z_6^2 + c_5(z)\, z_1^2\, z_3^2\, z_5^2$$

- the parameters ξ and η:

$$\frac{\xi(z)}{27} = \frac{f_6(z)}{f_2^3(z)} \qquad \text{and} \qquad \frac{\eta(z)}{4} = \frac{f_4(z)}{f_2^2(z)}$$

$$f_2(z) = c_0(z)\,, \qquad f_4(z) = c_4(z)\, c_5(z)\,, \qquad f_6(z) = c_1(z)\, c_2(z)\, c_3(z)\,.$$

- the brane content:

	f_6	f_4
$(q_1,\, q_2)$	$(1,1)$	$(0,3)$

- local tadpole cancellation for σ, τ constant over the whole complex plane:

$$f_4(z) = f_2(z)^2 \qquad \text{and} \qquad f_6(z) = f_2(z)^3$$

$\rightarrow$ the zeros of $f_2(z)$ can be interpreted as the positions of the O-planes

The $\eta(z) = 0$ limit (set $f_4(z) = c_4(z)\, c_5(z)$ to 0):

- $j_2 \to \infty$, i.e. $\tau^{(2)} \to i\,\infty$.
- if $\tau^{(2)} = C_0 + i\, e^{-\phi}$: weak coupling limit
- if $\tau^{(2)} = C_4 + i\, \mathrm{Vol}(T^4)$: large volume-limit
- in this limit we have:

$$\tau^{(1)} \;=\; \frac{\varpi_{10}}{\varpi_{00}} \;=\; \frac{i}{\sqrt{3}}\, \frac{{}_2F_1\left(\frac{1}{3},\frac{2}{3};1;1-\xi\right)}{{}_2F_1\left(\frac{1}{3},\frac{2}{3};1;\xi\right)}$$

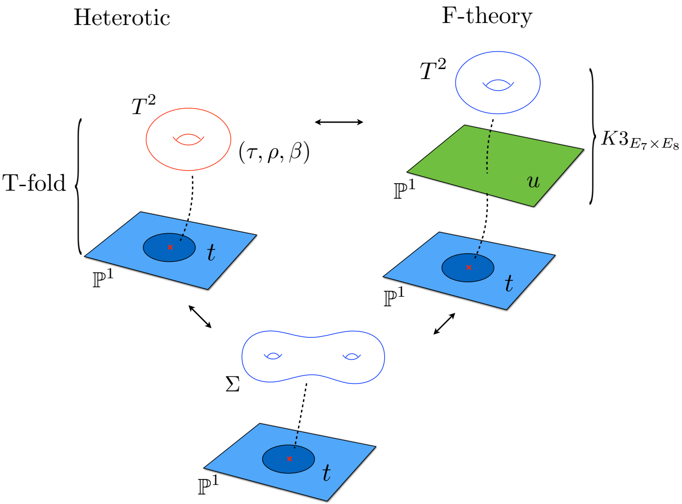
$$j_1 = j(\tau_1) = \frac{3^3(1+8\xi)^3}{\xi(1-\xi)^3}\;.$$

- $\xi(z)$ has degree 6 $\to$ 24 poles for j_1.
- j_1 can be identified with the j-invariant for an elliptic curve contained inside the $K3$ fiber.

Open questions:

- What is the interpretation of the auxiliary geometry? Is it more than simply specifying the monodromies? Are there other dualities in this picture?
- Generically, the solutions correspond to non-geometric U-folds from the ten-dimensional perspective, since the metric of the internal space, and other supergravity fields, are in general patched up by non-geometric U-duality transformations.
- What is the holographic interpretation of these backgrounds with varying fluxes?

Thank you!



[Font, Garcia-Etxebarria, Lüst, Massai, Mayrhofer, 2016]