Heterotic *Spin*(7) and 1/4-BPS solutions: Theory and Applications

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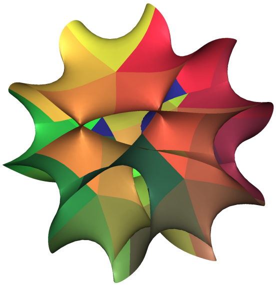
Outline

1. **Background: heterotic strings and SU(3) structure**
   - Moduli stabilization overview
   - SU(3) structure and half-flat manifolds
   - Domain wall vacuum
2. *Spin*(7) **structures**
   - *Spin*(7) and generalized half-flat manifolds
   - 10*d* flow equations and matching to 4*d* 1/4-BPS equations
3. **Applications**
   - 1/4-BPS solutions
   - Inflation?

Calabi–Yau compactification

- Superstring theory is self-consistent only in 10 spacetime dimensions.
- Assume the extra 6 spatial dimensions are compactified.
- Lots of supersymmetry in $d = 10$ $\rightarrow$ want to break most of it.
- Amount of broken SUSY $\Rightarrow$ holonomy group of compactification manifold.



- Maximum holonomy is SO(6) $\cong$ SU(4) $\Rightarrow$ no SUSY preserved.
- Calabi–Yau manifold: SU(3) holonomy $\Rightarrow$ 1/4 SUSY preserved
  - e.g. heterotic Calabi-Yau: 4 of 16 supercharges unbroken.

Problems with heterotic moduli stabilization

- In heterotic string theory, only have NS-NS flux H_3.
- Can stabilize complex structure moduli... then what?
- Dilaton can be stabilized by gaugino condensation.
- No other non-perturbative effects, no other options for flux quantization.
- In fact, problem is even worse:

Strominger, 1986

If a heterotic compactification on a manifold Y has a maximally symmetric (e.g. Poincaré) vacuum and non-vanishing H_3, Y is non-Calabi–Yau.

- Hence for a Calabi–Yau compactification, $H_3 = 0$!

What is an SU(3) structure manifold?

- Mirror dual of H_3: manifold with SU(3) structure hep-th/0008142 (Vafa), hep-th/0211102 (Gurrieri *et al*).
- SU(3) structure: there is a globally-defined spinor ζ that leaves 1/4 of the SUSY unbroken.
- Calabi–Yau case: ζ is covariantly constant with respect to the Levi-Civita connection ∇.
- Non-CY case: $\nabla\zeta \sim T^0\zeta$ (note: Γ matrices/indices suppressed).
- T^0 is the intrinsic torsion of the manifold.
- SU(3) decomposition: torsion splits into 5 torsion classes,

$$T^0 \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4 \oplus \mathcal{W}_5 \ .$$

Half-flat manifolds

Two (not mutually exclusive) ways to satisfy Strominger's theorem:

Option 1:

Study compactifications on SU(3) structure manifolds with torsion.

- Has been studied in eg. hep-th/0408121 (Gurrieri, Lukas, Micu), hep-th/0507173 (de Carlos, Gurrieri, Lukas, Micu).
- Torsion quantization understood for half-flat manifolds.
- Expanding the SU(3) invariant forms on appropriate bases, the only non-closed basis forms in the half-flat case satisfy

$$d\omega_i = e_i \beta^0 \,, \quad d\alpha_0 = e_i \tilde{\omega}^i \,.$$

- For half-flat manifolds, torsion falls into the SU(3) classes

$$T^0 \in \mathcal{W}_1^+ \oplus \mathcal{W}_2^+ \oplus \mathcal{W}_3 \,,$$

where $+$ denotes the real part of $\mathcal{W}$.

Domain wall vacuum

Option 2:

Break maximal symmetry of $d = 4$ spacetime.

- Compactification with H-flux on a half-flat or Calabi-Yau manifold.
- There exist 1/2-BPS domain wall solutions 1305.0594 (Klaput, Lukas, Svanes).
- 1/2-BPS: 2 of the 4 SUSY generators in $d = 4, \mathcal{N} = 1$ unbroken.
- $d = (2+1)$ Poincaré symmetry preserved; DW breaks symmetry in transverse y direction.
- Moduli satisfy flow equations in the y coordinate.
- 10d perspective: SU(3) fibred over $y \to G_2$ structure arXiv:1005.5302 (Lukas, Matti).

Spin(7): two transverse coordinates

- The domain wall solution is a special case of the metric

$$ds_4^2 = e^{-2B(x_a)}\left(\eta_{\alpha\beta} d\tilde{x}^\alpha d\tilde{x}^\beta + g_{ab} dx^a dx^b\right) .$$

- We can consider more general codimension-2 topological defects.
- 10d perspective: looks like an 8-dimensional *Spin*(7) structure.
- For the corresponding 6*d* compact SU(3) structure manifold, consider a generalized half-flat manifold, which satisfies

$$d\omega_i = p_{Ai}\beta^A - q_i^A \alpha_A , \quad d\alpha_A = p_{Ai}\tilde{\omega}^i , \quad d\beta^A = q_i^A \tilde{\omega}^i , \quad d\tilde{\omega}^i = 0 ,$$

  where ω_i and (α_A, β^B) are basis 2- and 3-forms, respectively.
- Relevant SU(3) torsion classes are now

$$T^0 \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 .$$

Flow equations: ten-dimensional perspective

- Ten-dimensional perspective:
  6 compact dimensions + 2 non-compact directions x and y
  $\rightarrow$ 8*d* *Spin*(7) structure.
- Killing spinor can be written in terms of invariant Cayley 4-form Ψ.
- Decompose Ψ under the 6*d* SU(3) structure as

$$\Psi = \mathrm{Re}(\mathrm{d}z \wedge \Omega) + \frac{1}{2} J \wedge J + \mathrm{dvol}_2 \wedge J \, ,$$

  where $\mathrm{d}z = \mathrm{d}x + i\mathrm{d}y$, $\mathrm{dvol}_2 = \mathrm{d}x \wedge \mathrm{d}y$.
- J is a 6*d* Kähler (1,1)-form, and Ω is a holomorphic (3,0)-form
  (for a Calabi–Yau they are harmonic, $\mathrm{d}J = \mathrm{d}\Omega = 0$).
- 10*d* supersymmetry transformations give the 8*d* flow equations,

$$*_8 \hat{H} = -e^{2\hat{\phi}} \mathrm{d}_8 (e^{-2\hat{\phi}} \Psi) \, , \quad 12 \mathrm{d}_8 \hat{\phi} = \Psi \lrcorner \mathrm{d}_8 \Psi = - *_8 (\Psi \wedge *_8 \mathrm{d}_8 \Psi) \, ,$$

  where $\hat{\phi}$ is the 10*d* dilaton and $\hat{H}$ is the 10*d* NS-NS flux.

Compactification

- Expand J and Ω in terms of the basis forms:

$$J = v^i \omega_i \ , \quad \Omega = \mathcal{Z}^A \alpha_A - \mathcal{G}_A \beta^A \ .$$

- The NS-NS 2-form potential and 3-form flux can be expanded as

$$\hat{B} = B + b^i \omega_i \ , \quad \hat{H} = H + \mathrm{d}b^i \wedge \omega_i + b^i \mathrm{d}\omega_i + H_{\mathrm{flux}} \ ,$$

  where we have introduced $H_{\mathrm{flux}} = \mu^A \alpha_A - \epsilon_A \beta^A$.
- Moduli superfields, including $4d$ dilaton ϕ:

$$S = a + i e^{-2\phi} \ , \quad T^i = b^i + i v^i \ , \quad Z^a \equiv \mathcal{Z}^a / \mathcal{Z}^0 = c^a + i w^a \ .$$

- Consider *H-flux only on the internal space* $\Rightarrow a \sim b^i \sim$ constant.
- Under SU(3)-structure decomposition, flow equations reduce to:

$$\mathrm{d}J = 2\mathrm{Im}\left(\partial_{\bar{z}}\Omega - 2\partial_{\bar{z}}\hat{\phi}\Omega\right) - *\hat{H} \ ;$$

$$\mathrm{d}\Omega = \partial_z (J \wedge J) - 2\partial_z \hat{\phi} J \wedge J \ ; \qquad \Omega \wedge \hat{H} = 4i * \partial_z \hat{\phi} \ .$$

Four-dimensional perspective

- Let us also consider an ansatz for the $4d$ theory.
- Unbroken supercharge in singlet of $Spin(7)$ $\Rightarrow$ 1/4-BPS in $4d$.
- 1/4-BPS ansatz: $\overline{\zeta} = \sigma^2\zeta = i\sigma^3\zeta$ gives Killing spinor equations

$$\begin{aligned}(\partial_x + i\partial_y)\, A^I &= -ie^{-B}e^{K/2}K^{IJ^*}D_{J*}W^* \ , \\ (\partial_x + i\partial_y)\, B &= -ie^{-B}e^{K/2}W^* \ , \\ 0 &= \mathrm{Im}(K_I\partial_a A^I) \ , \\ 2\partial_a \zeta &= -\partial_a B\zeta \ ,\end{aligned}$$

  where $a \subset \{x, y\} \equiv \{2, 3\}$, and $A^I = (S, T^i, Z^a)$.
- GVW superpotential (for generalized half-flat manifolds):

$$W = \sqrt{8}\int \Omega \wedge (\hat{H} + i\mathrm{d}J) = \sqrt{8}\,(\tilde{\mu}^A \mathcal{G}_A - \tilde{\epsilon}_A \mathcal{Z}^A) \ ,$$

  with modified flux parameters $\tilde{\epsilon}_A \equiv \epsilon_A - T^i p_{Ai}$, $\tilde{\mu}^A \equiv \mu^A - T^i q_i^A$.

Matching the 10*d* and 4*d* equations

- Consistency: need to match the 10*d* flow equations to the 4*d* 1/4-BPS Killing spinor equations 1512.02812 (SA, Matti, Svanes).

Summary of key points:

- 4*d* dilaton equation $\Rightarrow$ warp factor $B = \phi$ (up to a constant).
- $\mathrm{d}\Omega$ equation in 10*d* $\rightarrow$ KSE in 4*d* for the Kahler moduli T^i.
- $\mathrm{d}J$ equation in 10*d* $\rightarrow$ KSE for complex structure moduli Z^a.
- $\mathrm{d}J \wedge \bar{\Omega}$ with $\Omega \wedge \hat{H}$ $\rightarrow$ dilaton equation, $2\partial_{\bar{z}}\phi = -ie^{-\phi}e^{K/2}W^*$, BUT only if we also impose the additional 10*d* constraint

$$\int \partial_z \Omega \wedge \bar{\Omega} = \int \Omega \wedge \partial_z \bar{\Omega} \,.$$

- Actually, $\partial_z \Omega = K_z \Omega + \chi_z^{(2,1)}$; we are free to choose K_z real.
- Reducing this constraint to 4*d* $\rightarrow$ axion constraint, $K_a \partial_z c^a = 0$.

Explicit 1/4-BPS solution

- To study phenomenology, first need to find explicit solutions.
- Simple case (half-flat): p_{0i} and ϵ_a only non-zero parameters.
- Choosing c^a constant, constraints in (new) y-direction reduce to

$$\frac{\partial_y \mathcal{K}}{\mathcal{K}} = \frac{\partial_y \tilde{\mathcal{K}}}{\tilde{\mathcal{K}}} = -6\partial_y \phi = \frac{9(p_{0i}b^i - \epsilon_a c^a)}{\sqrt{\mathcal{K}\tilde{\mathcal{K}}}} \equiv \frac{A}{\sqrt{\mathcal{K}\tilde{\mathcal{K}}}} ,$$

  while the x-dependence is the same as for a domain wall.
- Here $\mathcal{K} = \mathcal{K}_{ijk} v^i v^j v^k$ and $\tilde{\mathcal{K}} = \tilde{\mathcal{K}}_{abc} w^a w^b w^c$.
- Solution:

$$\mathcal{K}(x, y) = \mathcal{K}_{\rm DW}(x) + A\mathcal{Z}^0 y , \quad \tilde{\mathcal{K}} = (\mathcal{Z}^0)^{-2}\mathcal{K} , \quad \phi = -\frac{1}{12} \ln\left(\mathcal{K}^2 \mathcal{K}_{\rm DW}\right) .$$

- For $A = 0$, reduces to 1/2-BPS domain wall (DW) solution.
- $A \neq 0$: additional divergences in y-direction — intersecting DW?

Inflation?

- Recall the generalized half-flat superpotential

$$W = \sqrt{8}\,(\tilde{\mu}^A \mathcal{G}_A - \tilde{\epsilon}_A \mathcal{Z}^A) = \sqrt{8}\,\left[(\mu^A - T^i q_i^A)\mathcal{G}_A - (\epsilon_A - T^i p_{Ai})\mathcal{Z}^A\right] .$$

- In non-geometric type-IIB compactifications, one can obtain superpotentials of the form

$$W = -(f_\lambda X^\lambda - \tilde{f}^\lambda F_\lambda) + iS(h_\lambda X^\lambda - \tilde{h}^\lambda F_\lambda) + iT_\alpha({q_\lambda}^\alpha X^\lambda - \tilde{q}^{\lambda\alpha} F_\lambda) ,$$

where X^λ and F_λ are period vectors.
- Such superpotentials were recently used to realize models of Starobinsky-like inflation (eg. Blumenhagen *et al*, 1503.01607).

Question:

Can similar inflation models also be constructed from generalized half-flat heterotic compactifications? (Topic for further research.)

Summary

- String compactifications generate moduli, which must be stabilized. This can be done using fluxes.
- For the heterotic string, only H_3 present. One can compactify on SU(3) structure manifolds which are not Calabi–Yau, and/or sacrifice maximal symmetry in $d = 4$. Domain wall solutions have been studied.
- We considered the more general codimension-2 case: from a Spin(7) ansatz compactified on generalized half-flat manifolds, the flow equations correspond to 1/4-BPS solutions in 4d.
- We found a possible 1/4-BPS intersecting domain wall solution.
- The superpotential for the generalized half-flat case is similar to that for non-geometric type IIB inflation models — work in progress.