String Duality in Multiply fibered CY_3

Xin Gao

Work with: L. Anderson, J. Gray, S. Lee,

Based on: arXiv: 1607.xxxxx, 1607.xxxxx

VirginiaTech Invent the Future®

24.June, 2016 @ String Pheno 16, Ioannina, Greece

Outline

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F-theory Compactification

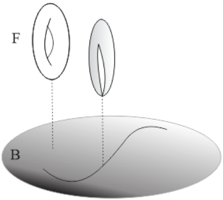
F-theory: geometric reinterpretation of orientifold IIB theory with (p, q) 7-brane and varying axion-dilaton τ (auxilary 2-torus).

Fibration on Calabi-Yau 3-fold (or 4-folds).

$$\pi : X_3 \xrightarrow{\mathbb{F}} B_2$$

If the fibration contains sections, X_3 can be written in Weierstrass form:

$$y^2 = x^3 + F(u)\, x\, z^4 + G(u)\, z^6$$



Singularity of elliptic fibers captured by discriminant

$$\Delta = 4F^3 + 27G^2 = 0$$

non-Abelian Gauge Group, Matter contents, Yukawa couplings . . .
captured by codim=1,2,3 singularities.

Today, focus on 6d theory (Compactified on $\mathbb{E}CY_3$).

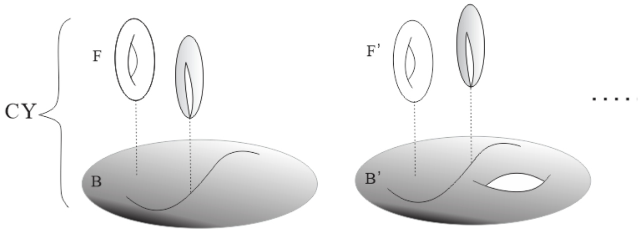
Multiple Fibrations

The fibration structure plays an important role in string duality :

- F-theory on K3 fibered n+1-folds ⇔ Heterotic on elliptic fibration n-folds
- F-theory on elliptic fibration n+1-folds ⇔ Type IIB on n-folds

Many manifolds not only contains one fibration, but multiple fibrations.

Multiple fibration ⇔ F theory duality ⇔ String duality zoo



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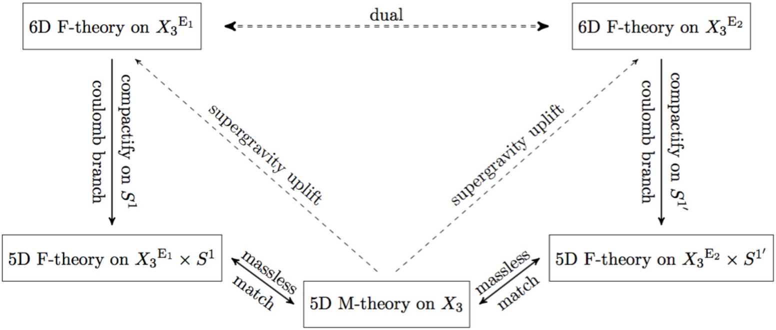
- Shared F-theory duality in 6D via M-theory in 5D



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Many manifolds not only have one fibrations, but multiple.

$$\text{Multiple fibration} \Leftrightarrow \text{F theory duality} \Leftrightarrow \text{String duality zoo}$$

- Shared F-theory duality in 6D via M-theory in 5D
- Het/F-theory in 4D/6D
- Het/Het duality: weak/weak or strong /weak

Questions

Q1: How to identify fibration structures in general? (James' talk)

Q2: Given an elliptic fibration structure, how to identify and construct sections explicitly?

Q3: How to relate the F-theory effective theory to arbitrary $\mathbb{E}CY_3$?

Q4: What the string duality through multiple fibrations looks like ?
(part of James' talk)

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Through one example of Shared F-theory duality in 6D

Outline

How to Identify the Section Topologically

The Section is mysterious.

- Holomorphic $s : B_2 \hookrightarrow X_3$. Rational $s' : B_2 \dashrightarrow B_2 \hookrightarrow X_3$
- The sections form an Abelian group called Mordell-Weil group

$$\# \, U(1)s = \mathrm{rk}(MW) \leq h^{1,1}(X_3) - h^{1,1}(B_2) - 1 - \mathrm{rk}\, G_{nA}$$

Some criteria for section $s \in \Gamma(X_3, \mathcal{O}_{X_3}(S))$:

- Oguiso criteria: $\forall \mathrm{pt.}\, p \in B_2,\ S \cdot F_p = 1$
- Birationality to the base: $S^2 \cdot D_\alpha = -[c_1(B_2)] \cdot S \cdot D_\alpha$ for $D_\alpha \in B_2$
- Euler number of section: $\chi(S) \geq \chi(B_2)$, "=" for holomorphic
- Cohomology criteria: $h^0(X_3, \mathcal{O}_{X_3}(S)) > 0$,
  h^0 count the # of global sections defined by linebundle $\mathcal{O}_{X_3}(S)$.

Anderson, Antoniadis, Borchmann, Braun, Braun, Collinucci, Cvetic, Donagi, Etxebarria, Grassi, Gray, Grimm, Keitel, Klevers, Kuntzler, Krippendorf, Klemm, Leontaris, Mayrhofer, Mayorga, Morrison, Oehlmann, Park, Palti, Piragua, Pena, Piragua, Ruhle, S-Nameki, Valandro, Weigand ...

Putative sections

$$K3 = \left[\begin{array}{c|cc} & P & Q \\ \hline \mathbb{P}^1_{\mathbf{x_1}} & 1 & 1 \\ \mathbb{P}^2_{\mathbf{x_2}} & 1 & 2 \\ \hline \mathbb{P}^1_{\mathbf{x_3}} & 1 & 1 \end{array}\right] \qquad \left[\begin{array}{c} S \\ \hline \alpha_1 \\ \alpha_2 \\ \hline \alpha_3 \end{array}\right]$$

$K3 = R(\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}) = \left\langle P^{(1,1,1)}_{\mathbf{x_1},\mathbf{x_2},\mathbf{x_3}},\ Q^{(1,2,1)}_{\mathbf{x_1},\mathbf{x_2},\mathbf{x_3}} \right\rangle$

Linebundle of the putative section: $[S] = \mathcal{O}_X(\alpha_1, \alpha_2, \alpha_3)$

- Oguiso: $S \cdot F_p = 1 :\ 2\alpha_1 + 3\alpha_2 = 1$
- Birationality: $2\alpha_1 + 3\alpha_2 = -\alpha_1(3\alpha_2 + 2\alpha_3) - \alpha_2(\alpha_2 + 3\alpha_3)$

$$\Longrightarrow \alpha_1 = -1 - 3k, \alpha_2 = 1 + 2k,\ \alpha_3 = 1 + 11k + 14k^2$$

- Euler: $-6\alpha_1\alpha_2 - 2\alpha_2^2 - 4\alpha_1\alpha_3 - 6\alpha_2\alpha_3 \geq \chi[B_1] = 2$
- Cohomology: $h^0(X_2, \mathcal{O}(S)) = 1$ for zero section

$$S_0 = \mathcal{O}_X(-1, 1, 1),\ S_1 = \mathcal{O}_X(2, -1, 4),\ S_3 = \ldots \text{ with } h^0(S_i) = 1$$

Q5: Are they really sections? Can we construct them explicitly?

Construct the Section and Group Law

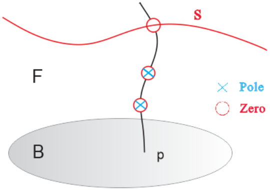
The divisor of section $S_0 = \mathcal{O}(-1,1,1)$ can be splitted as two parts

- Divisor of pole $[S_P] = \mathcal{O}(1,0,0)$
- Divisor of zero $[S_Z] = \mathcal{O}(0,1,1)$

$\forall\, p \in B$, $F_p \cdot S_P = 2$ while $F_p \cdot S_Z = 3$, s.t $F_p \cdot S = 1$.

Rational function for section s:

$$s = \frac{N[\mathbf{x_2}, \mathbf{x_3}]}{D[\mathbf{x_1}]}$$



Regular Condition: N vanish at the same order when D goes to zero on hypersurface

Matching the zeros of N and $D \Longrightarrow s_0 \in \Gamma(X, \mathcal{O}_X(S_0))$.

The free parameters in s count the # of sections. Here $h^0(X, \mathcal{O}_X(S_0)) = 1$.

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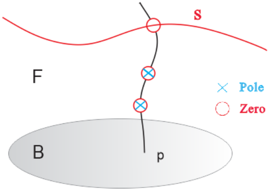

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The free parameters in s count the # of sections. Here $h^0(X, \mathcal{O}_X(S_0)) = 1$.

Choosing S_1 as generators of Mordell-Weil, get all other sections S_i:

$$\mathrm{Div}(ks_1) = S_0 + k(S_1 - S_0) + k(k-1)\pi(S_0 \cdot (S_1 - S_0))$$

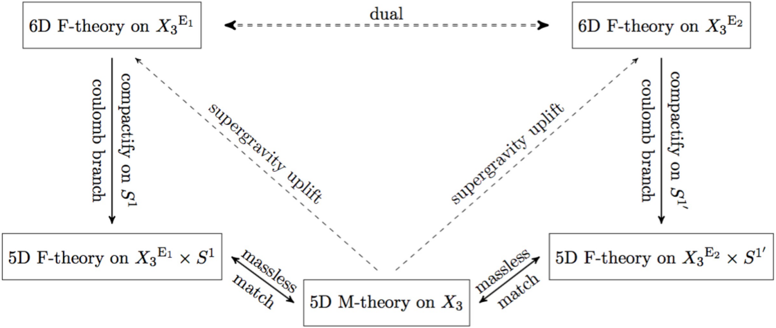
Outline

Multiple Fibrations

F-theory on one geometry $h^{1,1}(X_3) = 4, h^{1,2}(X_3) = 47$ with 4 diff fibrations:

$$X_3^{E_1} = \left[\begin{array}{c|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^2 & 1 & 0 & 2 & 0 \\ \mathbb{P}^2 & 0 & 1 & 1 & 1 \\ \mathbb{P}^2 & 1 & 0 & 1 & 1 \end{array}\right], \quad X_3^{E_2} = \left[\begin{array}{c|cccc} \mathbb{P}^2 & 0 & 1 & 2 & 0 \\ \mathbb{P}^2 & 0 & 1 & 1 & 1 \\ \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^2 & 1 & 0 & 1 & 1 \end{array}\right],$$

$$X_3^{E_3} = \left[\begin{array}{c|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^2 & 1 & 0 & 2 & 0 \\ \mathbb{P}^2 & 1 & 0 & 1 & 1 \\ \mathbb{P}^2 & 0 & 1 & 1 & 1 \end{array}\right], \quad X_3^{E_4} = \left[\begin{array}{c|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^2 & 0 & 1 & 1 & 1 \\ \mathbb{P}^2 & 1 & 0 & 1 & 1 \\ \mathbb{P}^2 & 1 & 0 & 2 & 1 \end{array}\right].$$



Singular Limit $\longrightarrow$ Weierstrass form

Singular Limit: push $X_3^{E_i}$ to get the Weierstrass form.

- Deligne procedure W(X).
- Jacobian procedure J(X).
- Minimal model procedure.

Singular Limit: Deligne procedure

$$X_3^{E_1} = \left[\begin{array}{c|cccc} \mathbb{P}^1_{\mathbf{x_1}} & 1 & 1 & 0 & 0 \\ \mathbb{P}^2_{\mathbf{x_2}} & 1 & 0 & 2 & 0 \\ \mathbb{P}^2_{\mathbf{x_3}} & 0 & 1 & 1 & 1 \\ \hline \mathbb{P}^2_{\mathbf{x_4}} & 1 & 0 & 1 & 1 \end{array}\right]$$

- Identify the zero section as $S = \mathcal{O}_{X_3}(-1,1,0,1)$. Identify and construct the coordinates Z, X, Y in Tate form as sections of $S, S^2 \otimes K_B^{-2}, S^3 \otimes K_B^{-3}$.

$$Z \in H^0(X_3, \mathcal{O}(-1,1,0,1)), \quad h^0(X_3, \mathcal{O}(Z)) = 1.$$
$$X \in H^0(X_3, \mathcal{O}(-2,2,0,8)), \quad h^0(X_3, \mathcal{O}(X)) = 29.$$
$$Y \in H^0(X_3, \mathcal{O}(-3,3,0,12)), \quad h^0(X_3, \mathcal{O}(Y)) = 66.$$

Singular Limit: Deligne procedure

$$X_3^{E_1} = \left[\begin{array}{c|cccc} \mathbb{P}^1{}_{\mathbf{x_1}} & 1 & 1 & 0 & 0 \\ \mathbb{P}^2{}_{\mathbf{x_2}} & 1 & 0 & 2 & 0 \\ \mathbb{P}^2{}_{\mathbf{x_3}} & 0 & 1 & 1 & 1 \\ \hline \mathbb{P}^2{}_{\mathbf{x_4}} & 1 & 0 & 1 & 1 \end{array}\right]$$

- Identify the zero section as $S = \mathcal{O}_{X_3}(-1,1,0,1)$. Identify and construct the coordinates Z, X, Y in Tate form as sections of $S, S^2 \otimes K_B^{-2}$, $S^3 \otimes K_B^{-3}$.

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- $H^0(X_3, \mathcal{O}(Y^2)) \ni X^3,\ B_0Y^2,\ B_1XYZ,\ B_2Z^2X^2,\ B_3Z^3Y,\ B_4Z^4X,\ B_5Z^6$
  $B_1 = \{B_{1,1}x_{4,0}^3 \ldots, B_{1,10}x_{4,2}^3\}, \ldots, B_5 = \{B_{5,1}x_{4,0}^{18}, \ldots, B_{5,190}x_{4,2}^{18}\}$

$\Longrightarrow$ 376 terms in the space $H^0(X_3, \mathcal{O}(Y^2))$ but the dim $h^0(X_3, \mathcal{O}(Y^2)) = 375$.

One linear relationship among 376 terms, which fix $\{B_{i,j}\} \Longrightarrow$ Tate form

- By coordinate rescaling:

$$y^2 + a_1xyz + a_3yz^3 = x^3 + a_2x^2z^2 + a_4xz^4 + a_6z^6$$
$$\Longrightarrow \Delta_w = I_2 \cdot I_1$$

Singular Limit: Jacobian procedure

$$X_3^{E_1} = \left[\begin{array}{c|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^2 & 1 & 0 & 2 & 0 \\ \mathbb{P}^2 & 0 & 1 & 1 & 1 \\ \mathbb{P}^2 & 1 & 0 & 1 & 1 \end{array}\right] \xrightarrow{\mathbb{P}^2} X_3'^{E_1} = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^2 & 1 & 2 \\ \mathbb{P}^2 & 1 & 2 \end{array}\right] \xrightarrow{\mathbb{P}^1} X_3''^{E_1} = \left[\begin{array}{c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array}\right]$$

- The codim 2 fiber in $X_3'^{E_1}$, we can directly calculate the F, G and Δ_{bl_1}. Volker, Grimm, Keitel, $\cdots$
- The codim 1 fiber in $X_3''^{E_1}$ we can apply the jacobian procedure of plane cubics to get F, G and Δ_{bl_2}. Artin, Villegas, Tate $\cdots$
- All the discriminants we get are consistent with the one Δ of $X_3^{E_1}$

$$\Delta \sim \Delta_w \cong \Delta_{bl_1} = \Delta_{bl_2} = I_2 \cdot I_1$$

- $G_{nA} = SU(2)$

Effective Theory: Gauge sector and Matter contents

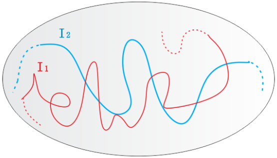
- From Shioda-Tate-Wazir fomula:

$$r = \mathrm{rk}\, MW = h^{1,1}(X_3) - h^{1,1}(B) - \mathrm{rk}G_{nA} - 1 = 1.$$
$$G_X = G_{nA} \times \prod U(1)^r = SU(2) \times U(1)$$

- $T = h^{1,1}(B) - 1 = 0$, $\quad V = \dim(\mathrm{adj}\,(G_{nA})) + r = 4$.
  $H = 277$, $\quad H_u = h^{2,1}(X_3) + 1 = 48$, $\quad H_c = 229$.

  charged matter: $22 \times \mathbf{2} + 185 \times \mathbf{1}$

$SU(2)$ doublets: Some codim 2 locus $I_2 \cap I_1$.
$U(1)$ charged matter:
Fully classify the singularities of I_1.



- 6D Gravity and non-Abelian anomaly cancelation works.

(non)-Abelian/Abelian Duality on X_3

$$X_3^{E_1} = \left[\begin{array}{c|cccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^2 & 1 & 0 & 2 & 0 \\ \mathbb{P}^2 & 0 & 1 & 1 & 1 \\ \hdashline \mathbb{P}^2 & 1 & 0 & 1 & 1 \end{array}\right], \quad X_3^{E_2} = \left[\begin{array}{c|c:ccc} \mathbb{P}^2 & 0 & 1 & 2 & 0 \\ \mathbb{P}^2 & 0 & 1 & 1 & 1 \\ \hdashline \mathbb{P}^1 & 1 & 1 & 0 & 0 \\ \mathbb{P}^2 & 1 & 0 & 1 & 1 \end{array}\right],$$

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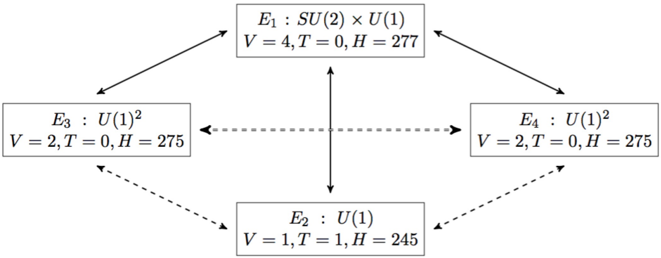
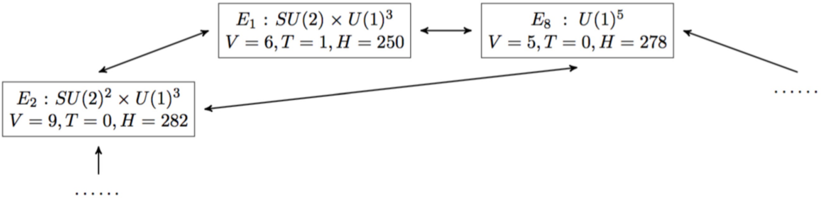


They share the same 5D M-theory (F-theory limit) with $V^M = 3, H^M = 48$.

non-Abelian Duality and Higher Rank of Modell-Weil

$$h^{1,1}(X_3') = 7,\ h^{2,1}(X_3') = 23. \quad {X'}_3^{E_1} = \left[\begin{array}{c|ccccccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ \mathbb{P}^2 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ \mathbb{P}^2 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ \hdashline \mathbb{P}^1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{array}\right],$$

$${X'}_3^{E_2} = \left[\begin{array}{c|ccccccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ \mathbb{P}^1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ \mathbb{P}^2 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ \mathbb{P}^2 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ \hdashline \mathbb{P}^2 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right], \quad \cdots \quad , {X'}_3^{E_8} = \left[\begin{array}{c|ccccccc} \mathbb{P}^1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ \mathbb{P}^1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \mathbb{P}^1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ \mathbb{P}^2 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ \mathbb{P}^2 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ \hdashline \mathbb{P}^2 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{array}\right]$$



Shared F-theory duality in 6D via the same 5D M-theory limit with

$$V^M = 6,\ H^M = 24.$$

Outline

Conclusion and Outlook

Conclusion:

- Establish a toolkit to analyze elliptic fibration with sections.
- Perform Deligne procedure and Jacobian procedure to get the Weieresstrass form for arbitary elliptic fibration explicitly.
- String duality in these multiple fibration CY_3. Like shared F-theory duality in 6D, Het/F-theory in 4D/6D and Het/Het duality.

Outlook:

- Math: Classify fibration strucutres and F-theory EFT in CICY 3-folds database. Extended to CY_4 or CY_5. Analyze the genus one curve and toric Calabi-Yau case.
- Physics: Determine the $U(1)$ charge explicitly. Search for more string duality in this framework. e.g, T-duality in LST.

Thanks for your attention!