Model Building on non-factorizable $T^6/(\mathbb{Z}_4 \times \mathcal{R}\Omega)$ Orientifolds

based on 1606.04926 with M. Berasaluce and G. Honecker

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Motivation

- So far only factorizable toroidal $\mathbb{Z}_4 \times \Omega\mathcal{R}$-orientifolds have been considered for model building and moduli stabilization in type IIA.

  Blumenhagen, Görlich, Ott 05, Ihl, Wrase 06 ...

- On the non-factorizable orientifolds, only models with the D6-branes on top of O6-planes were discussed.

  Blumenhagen, Conlon, Suruliz 04

- Aim: extend the analysis and construct models with arbitrary D6-branes.
- There are two non-factorizable $\mathbb{Z}_4$-orbifolds with lattices $A_3 \times A_3$ and $A_3 \times A_1 \times B_2$.

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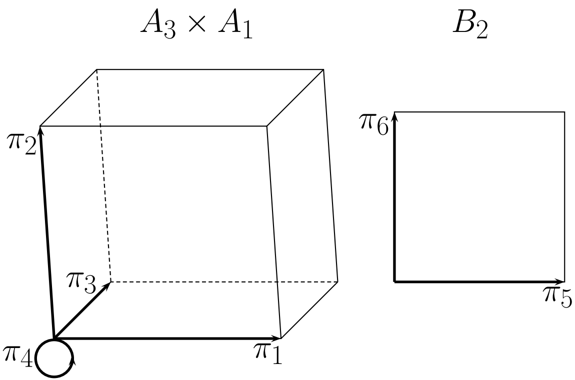
Preliminaries: $\mathbb{Z}_4$-orbifold with torus lattice $A_3 \times A_1 \times B_2$



$\mathbb{Z}_4$-action Q:

$$
\begin{aligned}
Q\pi_1 &= \pi_2 \\
Q\pi_2 &= \pi_3\,, \\
Q\pi_3 &= -\pi_1 - \pi_2 - \pi_3\,, \\
Q\pi_4 &= -\pi_4\,, \\
Q\pi_5 &= \pi_6\,, \\
Q\pi_6 &= -\pi_5
\end{aligned}
$$

- Consider type IIA compactified on $T^6/\mathbb{Z}_4$-orbifold
- Q preserves $\mathcal{N} = 2$ SUSY in 4 dim
- Q fixes the metric of the six-torus up to 3 radii and 4 angles
- Space-time filling D6-branes wrapping 3-cycles on the internal space

Bulk 3-cycles

- Ansatz: any toroidal 3-cycle described by 10 wrapping numbers

$$\pi^{\text{torus}} := \bigwedge_{i=1}^{2} (m^i \pi_1 + n^i \pi_2 + p^i \pi_3 + q^i \pi_4) \wedge (m^3 \pi_5 + n^3 \pi_6)$$

- By taking orbits of the Q-action ($\tilde{Q} := \sum_{n=0}^{3} Q^n$), a basis of $\mathbb{Z}_4$-invariant *bulk* 3-cycles is given by

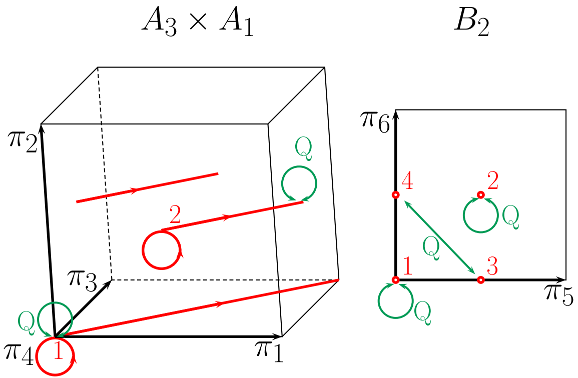
$$\gamma_1 := \tilde{Q}\pi_{135} \quad \gamma_2 := -\tilde{Q}(\pi_{125} + \pi_{126}), \quad \bar{\gamma}_1 := -\tilde{Q}\pi_{145}, \quad \bar{\gamma}_2 := -\tilde{Q}\pi_{245}$$

- Any bulk cycle π^{bulk} corresponding to π^{torus} can be decomposed in this basis

$$\begin{aligned}\pi^{\text{bulk}} = \tilde{Q}\pi^{\text{torus}} =& \left[\frac{2A_2 - A_1 - A_3}{2} m^3 + \frac{A_1 - A_3}{2} n^3\right]\gamma_1 + \left[(B_3 - B_1)m^3 + B_2 n^3\right]\bar{\gamma}_1 \\ & \left[\frac{A_3 - A_1}{2} m^3 + \frac{2A_2 - A_1 - A_3}{2} n^3\right]\gamma_2 + \left[-B_2 m^3 + (B_3 - B_1) n^3\right]\bar{\gamma}_2\end{aligned}$$

with $A_i, B_i \in \mathbb{Z}$

Exceptional 3-cycles



- $\mathbb{Z}_2$-invariant 1-cycles $\pi_1 + \pi_3$, π_4 and exceptional 2-cycles e_{ij} $\Rightarrow$ exceptional 3-cycles:

$$\gamma_3 := (e_{13} - e_{14}) \wedge (\pi_1 + \pi_3)\,, \qquad \bar{\gamma}_3 := (e_{13} - e_{14}) \wedge \pi_4\,,$$
$$\gamma_4 := (e_{23} - e_{24}) \wedge (\pi_1 + \pi_3)\,, \qquad \bar{\gamma}_4 := (e_{23} - e_{24}) \wedge \pi_4\,.$$

- Non-vanishing intersection numbers

$$\gamma_i \circ \bar{\gamma}_j = -2\delta_{ij} \quad i,j = 1,2, \qquad \gamma_m \circ \bar{\gamma}_n = 2\delta_{mn} \qquad m,n = 3,4.$$

Fractional 3-cycles

- Intersection form is not unimodular. Search for better basis $\Rightarrow$ fractional 3-cycles: $\pi^{\text{frac}} = \frac{1}{2}\pi^{\text{bulk}} + \frac{1}{2}\pi^{\text{exc}}$
- Need $\mathbb{Z}_2$-invariant toroidal 3-cycles $\Rightarrow$ constraints on the wrapping numbers

$$Q^2\pi^{\text{torus}} = \pi^{\text{torus}} \Leftrightarrow A_1 + A_3 = 0 \text{ and } B_1 - B_2 + B_3 = 0$$

- Only such 3-cycles can split into fractional cycles

$$\pi^{\text{frac}} = \frac{1}{2}\pi^{\text{bulk}} + \frac{N}{2}(s^1\tau^1\gamma_3 + s^2\tau^2\gamma_4) + \frac{\bar{N}}{2}(s^1\tau^1\bar{\gamma}_3 + s^2\tau^2\bar{\gamma}_4)$$

Fractional 3-cycles

- New basis of $H_3(T^6/\mathbb{Z}_4, \mathbb{Z})$

$$\alpha_1 := \frac{\gamma_2-\gamma_3}{2}, \quad \alpha_2 := \frac{\gamma_3-\gamma_4}{2}, \quad \alpha_3 := \gamma_4, \quad \alpha_4 := \frac{\gamma_1-\gamma_2-\gamma_3-\gamma_4}{2}$$
$$\bar{\alpha}_1 := \bar{\gamma}_2 + \bar{\gamma}_3, \quad \bar{\alpha}_2 := -\bar{\gamma}_3 + \bar{\gamma}_4, \quad \bar{\alpha}_3 := -\bar{\gamma}_4, \quad \bar{\alpha}_4 := \frac{\bar{\gamma}_1-\bar{\gamma}_2+\bar{\gamma}_3+\bar{\gamma}_4}{2}$$

- Intersection form $\begin{pmatrix} 0 & C_{F_4} \\ -C_{F_4} & 0 \end{pmatrix}$
- Choose new basis on $A_3 \times A_1$-torus

$$v_1 := \pi_1 + \pi_2, \quad v_3 := \pi_1 + \pi_3,$$
$$v_2 := \pi_2 + \pi_3, \quad v_4 := \pi_4$$

- Any $\mathbb{Z}_2$-invariant 3-cycle can be described by 6 wrapping numbers
- $\mathbb{Z}_4$-orbifold with $A_3 \times A_1 \times B_2$-lattice $\Rightarrow$ factorized $\mathbb{Z}_4$-orbifold with $B_2 \times B_2 \times A_1 \times A_1$-lattice + shift symmetry

Blaszczyk, Groot Nibbelink, Ruehle 12

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Orientifolding

- Complex coordinates on $T^6/\mathbb{Z}_4$

$$z^1 := \frac{1}{\sqrt{2}}(x^1 + ix^2 - x^3), \quad z^3 := x^5 + ix^6,$$
$$z^2 := \frac{1}{2\sqrt{2u_2}}(x^1 - x^2 + x^3 + 2\mathcal{U}x^4)$$

  with the complex structure $\mathcal{U} := u_1 + iu_2$
- The orientifold projection $\Omega\mathcal{R}$ breaks SUSY to $\mathcal{N} = 1$
- $\mathcal{R}$ is an antiholomorphic involution $\mathcal{R}: z^k \mapsto e^{i\theta_k}\bar{z}^k$
- There are four possible choices for θ

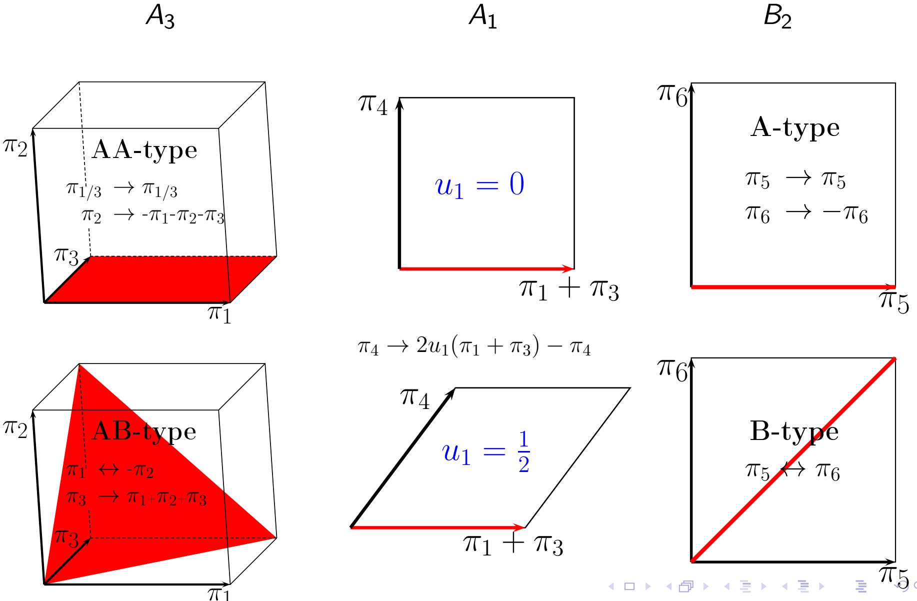
$$(0,0,0) \to \textbf{AAA}, \quad (0,0,\pi/2) \to \textbf{AAB}$$
$$(\pi/2,0,0) \to \textbf{ABA}, \quad (\pi/2,0,\pi/2) \to \textbf{ABB}$$

the lattice notation is taken from Blumenhagen, Conlon, Suruliz 04

- Every choice of θ gives rise to two orientations of A_1 relative to A_3
  $\Rightarrow$ $u_1 = 0$ or new $\frac{1}{2}$

Orientifolding

$\mathcal{R}$ acts on the 1-cycles π_i



Orientifolding

- Relation between the orientation of the projections axes
  $\Rightarrow$ 4 independent orientifolds

$$\left.\begin{array}{lcl} \textbf{AAA} \text{ with } u_2 & \equiv & \textbf{AAB} \text{ with } \frac{1}{2u_2} \\ \textbf{ABA} \text{ with } u_2 & \equiv & \textbf{ABB} \text{ with } \frac{1}{2u_2} \end{array}\right\} \text{for } u_1 = 0$$

$$\left.\begin{array}{lcl} \textbf{AAA} \text{ with } u_2 & \equiv & \textbf{ABB} \text{ with } \frac{1}{4u_2} \\ \textbf{ABA} \text{ with } u_2 & \equiv & \textbf{AAB} \text{ with } \frac{1}{4u_2} \end{array}\right\} \text{for } u_1 = \frac{1}{2}$$

Semi-realistic models

- The O6-planes are given by the fixed point loci of the involution.
- The presence of O6-planes enables the fulfilment of the RR-tadpole cancellation conditions $\Rightarrow$ global models
- We can reproduce ($u_1 = 0$) and extend ($u_1 = \frac{1}{2}$) the results with D6-branes on top of the O6-plane

Blumenhagen, Conlon, Suruliz 04

		$u_1 = 0$	$u_1 = \frac{1}{2}$
lattice	θ	$U(M) \times U(N)$	$U(M) \times U(N)$
AAA	$(0,0,0)$	$U(8) \times U(4)$	$U(8) \times U(2)$
AAB	$(0,0,\frac{\pi}{2})$	$U(4) \times U(8)$	$U(4) \times U(4)$
ABA	$(\frac{\pi}{2},0,0)$	$U(8) \times U(4)$	$U(4) \times U(4)$
ABB	$(\frac{\pi}{2},0,\frac{\pi}{2})$	$U(4) \times U(8)$	$U(2) \times U(8)$

The *sLag* conditions $\longleftrightarrow$ SUSY of 3-cycles

- *sLag* (= special Lagrangian) conditions $\longleftrightarrow$ SUSY conditions:

$$J_2\big|_\pi = 0 \quad \textit{Lag} \text{ condition}$$

- not every 3-cycle is *Lag* (on non-factorizable orbifold)
- any $\mathbb{Z}_2$-invariant 3-cycle is *Lag* $\rightarrow$ any fractional 3-cycle is *Lag*

$$\left.\begin{array}{l} \mathrm{Im}(e^{i\varphi_a}\Omega_3)\big|_\pi = 0 \\ \mathrm{Re}(e^{i\varphi_a}\Omega_3)\big|_\pi > 0 \quad \text{calibration cond.} \end{array}\right\} \Rightarrow \textit{sLag} \text{ cond's}$$

$\rightarrow$ constraints regarding wrapping numbers and complex structure modulus

Semi-realistic models

- We are able to construct some $\mathcal{N} = 1$ SUSY Pati-Salam models with intersecting D6-branes
- There are 2 and 4 generation global SUSY PS models
- With 4 generations; only $U(4)_a \times USp/SO(2)_b \times USp/SO(2)_c$-models do not have (anti-)symmetric representations on b- and c- stack

Chiral spectrum of a global PS model with 4 generations on **ABB** (for $u_1 = 0$)	
sector	$SU(4)_a \times USp/SO(2)_b \times USp/SO(2)_c \times SU(4)_{h_1} \times USp/SO(2)_{h_2} \times U(1)^2$
$ab = ab'$	$4 \times (4, \bar{2}, 1, 1, 1)_{(1,0)}$
$ac = ac'$	$4 \times (\bar{4}, 1, 2, 1, 1)_{(-1,0)}$

- Some exotic local SUSY PS-models with 3 generations are possible on **ABB**-lattice

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Summary and Outlook

Summary

- Study of the geometry of toroidal $\mathbb{Z}_4$-orbifolds of the type $A_3 \times A_1 \times B_2$ and $A_3 \times A_3$
  $\Rightarrow$ differences between the non- and factorizable orbifolds
- Characterization of possible orientifold projections
- Construction of some semi-realistic global supersymmetric PS models

Outlook

- Extend the search for global three-generation semi-realistic models
- Classification of the type of symmetry enhancement to USp- or SO-gauge groups
- Extend the analysis to other non-factorizable orientifolds

Thanks for your attention!