The correspondence between orbifolds and free fermionic models and applications to phenomenology

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Motivation

- String theory provides the most promising framework for a fundamental theory of physics.
- Many semi-realistic heterotic string models have been constructed.

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The landscape problem

Motivation

- There have been extensive computer scans towards that goal, both in the (bosonic) orbifold and in the free fermionic formulation.
  Fischer, Ratz, Torrado, Vaudrevange 2013, Faraggi, Rizos, Sonmez 2014, ...
- It would be useful to have a dictionary between the two formalisms to compare these results.

Previous work

Various aspects of the correspondence have been discussed in the past
Kiritsis, Kounnas '97, Gregori, Kounnas, Rizos '99, Donagi, Faraggi '04, Donagi, Wendland '08

However, a complete model builder's dictionary that would allow for a computational comparison was not available.

Orbifold models

We are interested in **toroidal orbifolds**. Such models are specified by:

1) A **Narain lattice** on which the internal 6 dimensions (and the gauge degrees of freedom) are compactified.

2) An **orbifold action** compatible with the lattice.

3) A choice of the relative phases when we have more than one action (**discrete torsion**).

Free fermionic models

In these models all the degrees of freedom needed to cancel the conformal anomaly are implemented as worldsheet fermions.

Free fermionic models are specified by:

1) A set of **basis vectors** that describe the boundary conditions of the worldsheet fermions around the cycles of the worldsheet torus.
2) A choice of the relative phases between different basis vectors (**discrete torsion**).

Converting from one to the other

To convert a free fermionic model to an orbifold we must know how to implement the following steps:

1) Extract the **Narain lattice** from the basis vectors.
2) Extract the **orbifold action** from the basis vectors.
3) Extract the orbifold phases (**discrete torsion**) from the free fermionic phases.

1) Extracting the Narain lattice

The geometric data of the orbifold model can be read from the untwisted part of the partition functions in the two formalisms, *i.e.*

$$\mathcal{Z}_{\text{FFF}} = \underbrace{\sum_{\alpha,\beta} C\left[{\alpha \atop \beta}\right] Z\left[{\alpha \atop \beta}\right]}$$

$$\downarrow$$

$$\mathcal{Z}_{\text{orbi}} = \mathcal{Z}_{\text{untwisted}}(G, B, A, g) + \mathcal{Z}_{\text{twisted}}$$

$$\|$$

$$\sum_{p_L, p_R} q^{\frac{1}{2} p_L(G,B,A,g)^2} \bar{q}^{\frac{1}{2} p_R(G,B,A,g)^2}$$

Note that this process yields specific values for the bosonic moduli (G, B, A). Such special points are called *free fermionic points*.

2) orbifold action from the basis vectors

Using the *bosonization/fermionization formula*

$$y + iw = :e^{iX}:$$

one might expect that:

- When
$$y + iw \to -(y + iw) \quad \Rightarrow \quad X \to X + \pi$$
(<u>shift</u> action)
- When
$$y + iw \to y - iw \quad \Rightarrow \quad X \to -X$$
(<u>twist</u> action)
- When
$$y + iw \to -y + iw \quad \Rightarrow \quad X \to -X + \pi$$
(<u>roto-translational</u> action)

2) orbifold action from the basis vectors

- However the previous statements are not coordinate independent.
- For $\mathbb{Z}_2 \times \mathbb{Z}_2$ actions it is the overlap between the two free fermionic basis vectors that really determines the orbifold action.

3) Extracting the discrete torsion

$$\mathcal{Z}_{\text{FFF}} = \sum_{\alpha,\beta} \quad C\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \quad Z\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

$$\updownarrow$$

$$\mathcal{Z}_{\text{orbi}} = \sum_{h,h':[h,h']=0} \quad C\begin{bmatrix}h\\ h'\end{bmatrix} \quad Z\begin{bmatrix}h\\ h'\end{bmatrix}$$

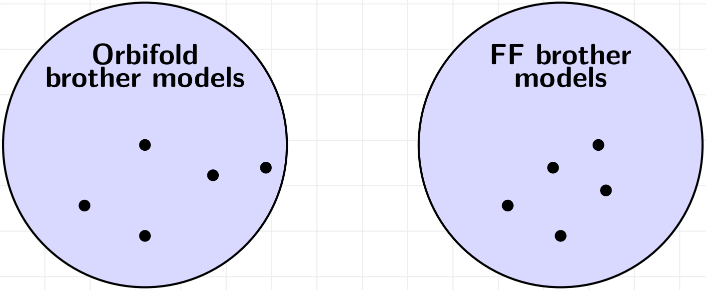
This should be a straightforward task...

3) Extracting the discrete torsion

Points to consider:

- There are phases that in one formalism are included in the C part and in the other in the Z part!
- The identification of phases depends on the exact algorithm for the identification of the orbifold action (step 2).

  We can see **mirage torsion** Ploger, Ramos-Sanchez, Ratz, Vaudrevange '07 on both sides:



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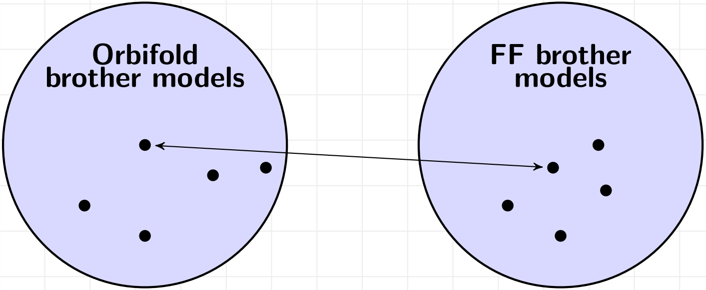

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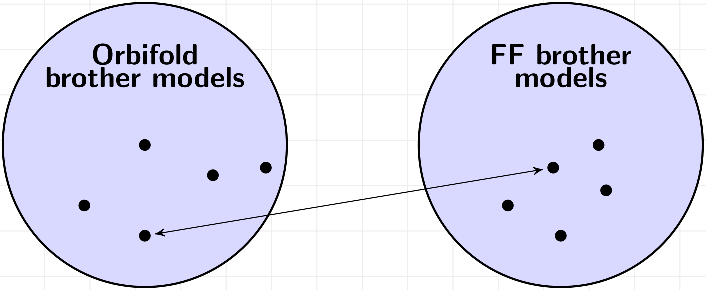

Open questions and applications

- The Narain moduli space needs to be understood better. For example, the generalized vielbein for such a model is of the form

$$E = \frac{1}{\sqrt{2}} \begin{pmatrix} \varepsilon + \varepsilon^{-T} C^T & -\varepsilon^{-T} & \varepsilon^{-T} A^T \alpha \\ \varepsilon - \varepsilon^{-T} C^T & \varepsilon^{-T} & -\varepsilon^{-T} A^T \alpha \\ \sqrt{2}\, A & 0 & \sqrt{2}\, \alpha \end{pmatrix} ,$$

  where $G = \varepsilon^T \varepsilon$ and $C = B + \frac{1}{2} A^T A$.

  However, if we are given such a lattice in an unknown basis, we are not aware of any general method that can bring the matrix to this form.

Open questions and applications

- The dictionary could be extended to asymmetric orbifolds. The technology we have developed allows as to easily handle asymmetric shifts, but asymmetric twists should be studied more carefully.
- It is much easier to construct asymmetric free fermionic models than orbifold models and it would be interesting to investigate what some of the realistic asymmetric free fermionic models correspond to.

Other (related) ideas

- The equivalence between bosons and fermions is neither new nor specific to the models we studied. A completely different recent application can be found in [Lokhande, Mukhi '15] where the equivalence was used to calculate Renyi and entanglement entropies.
- The moral of the story is that whenever someone is doing a computation involving bosons or fermions it is always worth asking what it would look like in the other formalism.

Summary and outlook

1. The heterotic string provides a nice framework to construct (semi-)realistic models. Understanding the **moduli space** of heterotic models is of great importance.
2. Free fermionic and orbifold models are related and we can translate from one to the other.
3. Such a dictionary also allows us to address difficult problems in one formalism using tools from the other.

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Thank you very much!