Geometric engineering of 2d (0,2) Theories via F-theory

1601.02015 (JHEP) with Sakura Schäfer-Nameki

+ in progress with SSN and Craig Lawrie

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(0,2) and String Theory

This talk is not directly on phenomenology, but it connects two key players of string phenomenology:

2d (0,2) gauge theories $\leftrightarrow$ **F-theory compactifications**

2d (0,2) theories define heterotic compactifications at the woldsheet level:

[Witten'93]

- UV (0,2) Gauged Linear Sigma Model $\xleftrightarrow[\text{RG}-\text{flow}]{}$ IR heterotic (0,2) SCFT
- In the geometric NLSM phase they define a target space geometry X_3 with vector bundle V_{het}:

$$\text{2d (0,2) theory} \xrightarrow[\text{string map}]{\text{heterotic}} \text{target space } (X_3, V_{\text{het}})$$

F-theory (0,2) engineering

This talk introduces a reversed process:

- We start with F-theory compactified on a torus-fibered Calabi-Yau 5-fold Y_5 with gauge flux G_4.
- The effective 2d theory is a (0,2) theory.
- Thus here the geometry (Y_5, G_4) defines a 2d (0,2) theory:

$$\text{compactification space } (Y_5, G_4) \xrightarrow{\mathrm{F-theory}} \text{2d (0,2) theory}$$

Extend geometric engineering in F-theory to 2d:

[Schafer-Nameki,TW'16]

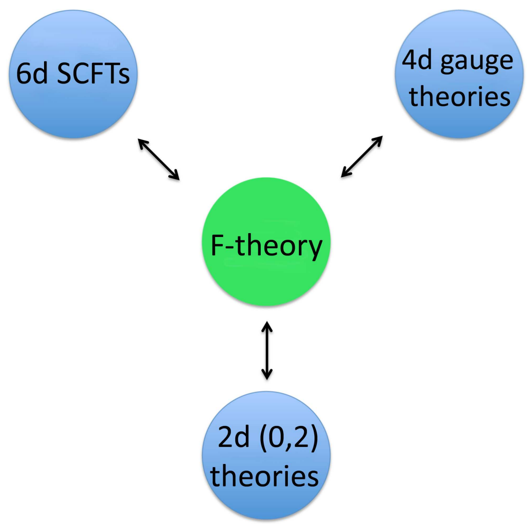
[Apruzzi,Hassler,Heckman,Melnikov'16]

D1 branes at CY4 sing.:

[Mohri'97] [Garcia-Compean,Uranga'98]

[Franco,Ghim,Lee,Seong,Yokoyama'15]

[Franco,Lee,Seong'15/16]



F-theory (0,2) engineering

Under suitable conditions the 2d (0,2) theory is a 2d (0,2) GLSM.

This inspires some interesting fantasies:

- Is there a correspondence

F-theory		heterotic
compactification geometry	$\overset{\longleftrightarrow}{?}$	target space
CY_5, 4-form flux		CY_3, heterotic bundle

- F-theory gives GLSM with general gauge groups

  - what is their heterotic interpretation?

  eg [Donagi,Sharpe'07],[Jockers,Kumar,Lapan,Morrison,Romo'12]

Does F-theory give a geometric interpretation of 2d (0,2) phenomena, e.g.

- non-abelian trialities? [Gadde,Gukov,Putrov'13]
- (0,2) mirror symmetry? e.g. [Blumenhagen et al'96] [Melnikov,Plesser'10] . . .
- interpretation of elliptic genus as computed in [Gadde,Gukov] [Benini,Eager,Hori,Tachikawa]'13; [Israel,Sarkis'15] ?

The setup

F-theory on elliptic (torus) fibration Y_5 over B_4:

effective theory in $\mathbb{R}^{1,1}$

$$\begin{array}{ccccc} \pi: & \mathbb{E}_\tau & \rightarrow & Y_5 \\ & & & \downarrow \\ & & & B_4 \end{array}$$

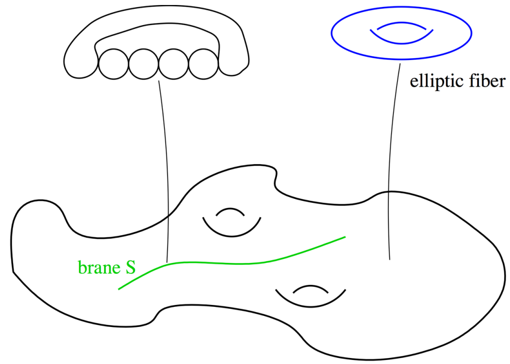
Gauge degrees of freedom $\leftrightarrow$ 7-branes on Kähler 3-cycle M_G

Two different regimes:

1. Decoupling limit:
   - Vol(B_4) $\to \infty$, but Vol(M_G) finite
   - 2d $N = (0,2)$ gauge theory $=$ (0,2) gauged linear sigma model (GLSM) with general gauge group from 7-branes
   - extra sector from D3-branes on complex curves
2. Coupling to 2d (0,2) supergravity:
   - Vol(B_4) finite
   - yet to be investigated in full detail

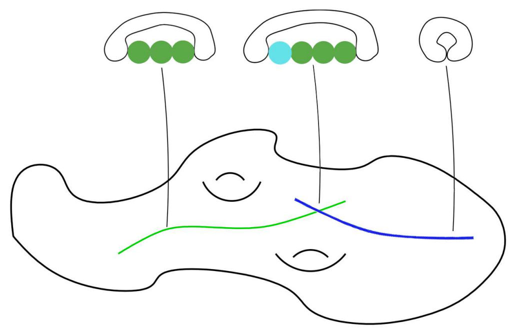
F-theory on CY5

7-brane on 3-cycle M_G — codimension one singularity — algebra $\mathfrak{g}$



$\mathbb{P}^1$s F_i above codim 1 loci M_G $\leftrightarrow$ Simple roots α_i of $\mathfrak{g}$

matter surface $S_{\mathbf{R}}$ — codimension two — fibre splitting



$\mathbb{P}^1$s above codim 2 loci $S_{\mathbf{R}}$ $\leftrightarrow$ Matter in Representation $\mathbf{R}$

$$F_i \rightarrow \underbrace{C_i^+}_{\text{weight of } \mathbf{R}} + \underbrace{C_{i+1}^-}_{\text{weight of } \bar{\mathbf{R}}}$$

F-theory on CY5

Questions:

1. What type of SUSY is preserved in 2d?
2. Which matter multiplets localize in codimension 1 and 2?
3. Which types of interactions are there, and in what codimension are they localised?

These questions can already be answered by reducing the 8d SYM theory on the 7-brane to $\mathbb{R}^{1,1} \times M_G$.

Twisting 8d SYM

- 8d SYM theory on a 7-brane with 16 SUSY parameters $\mathbf{8^s}_{-1} + \mathbf{8^c}_1$
- Worldvolume: $\mathbb{R}^{1,1} \times M_G$ $\qquad$ M_G: Kähler 3-cycle
- Holonomy group of M_G: $SO(6)_L \supset U(3)_L = SU(3)_L \times U(1)_L$

$$\mathbf{8^c}_{+1} \longrightarrow [\mathbf{4} + \bar{\mathbf{4}}]_1 \longrightarrow [(\mathbf{3}_{-1} + \mathbf{1}_3) + (\bar{\mathbf{3}}_1 + \mathbf{1}_{-3})]_1$$

- Supersymmetry $\longleftrightarrow$ globally defined spinor (holonomy singlet)
- **Topological Twisting** $=$
  redefine tangent bundle such that holonomy singlet exists [Vafa,Witten'94]

  Here:

$$U(1)_L \to U(1)_{\text{twist}} = \frac{1}{2}(U(1)_L + 3U(1)_R)$$

Twisting 8d SYM

- 8d SYM theory on a 7-brane with 16 SUSY parameters $\mathbf{8^s}_{-1} + \mathbf{8^c}_{+1}$

$$SO(1,7)_L \times U(1)_R \quad \rightarrow \quad SU(3)_L \times SO(1,1)_L \times (U(1)_L \times U(1)_R)$$

$$\begin{array}{ccc} \mathbf{8^c}_{+1} & \rightarrow & \mathbf{1}_{1;3,1} \oplus \mathbf{1}_{-1;-3,1} \oplus \mathbf{3}_{1;-1,1} \oplus \overline{\mathbf{3}}_{-1;1,1} \\ \mathbf{8^s}_{-1} & \rightarrow & \mathbf{1}_{-1;3,-1} \oplus \mathbf{1}_{1;-3,-1} \oplus \mathbf{3}_{-1;-1,-1} \oplus \overline{\mathbf{3}}_{1;1,-1} \end{array}$$

- Twisting:

$$U(1)_L \rightarrow U(1)_{\text{twist}} = \tfrac{1}{2}(U(1)_L + 3U(1)_R)$$

$$\bar{\epsilon}_- = \mathbf{1}_{-1;-3,1;0_{\text{twist}}}\,, \qquad \epsilon_- = \mathbf{1}_{-1;3,-1;0_{\text{twist}}}$$

$$\Longrightarrow \textbf{2d } N = (0,2) \textbf{ supersymmetry}$$

Reminder: 2d (0,2) SUSY

- 2 chiral supercharges with SUSY parameters ϵ_-, $\bar{\epsilon}_-$ [Witten'93]
- Superspace $(y^0, y^1; \theta^+, \bar{\theta}^+)$
- Matter content

Multiplet	Superfield	Content
gauge	V	$(v_0, v_1; \eta_-, \bar{\eta}_-; \mathfrak{D})$
chiral	Φ	$(\varphi; \chi_+; -)$
Fermi	P	$(-; \rho_-; E, G)$

- 2d gauge potential has no propagating degrees of freedom, but gauge symmetry acts as constraint
- Opposite chiralities: ρ_- versus χ_+
- Fermi auxiliary fields: G and $E = E(\Phi)$ holomorphic

7-brane Bulk Matter

Challenge: No string quantization in F-theory background known

Way out: Massless bulk matter descends from 8d SYM upon topological twisting cf 4d: [Beasley,Heckman,Vafa] [Donagi,Wijnholt]'08

- Bulk SYM matter on M_G: $\mathbf{8^v}_0$, $\mathbf{1}_{\pm 2}$, $\mathbf{8^c}_{+1}$, $\mathbf{8^s}_{-1}$
- Decompose w.r.t. $SO(1,7)_L \times U(1)_R \to SO(1,1)_L \times SU(3)_L \times (U(1)_L \times U(1)_R)$
- Internal component w.r.t. $SU(3)_L \times U(1)_{\mathrm{tw}}$: $U(1)_{\mathrm{tw}} = \frac{1}{2}(U(1)_L + 3U(1)_R)$

$\mathbf{1}_0 \leftrightarrow \Omega^{0,0}(M_G)$ $\mathbf{3}_1 \leftrightarrow \Omega^{0,1}(M_G)$ $\mathbf{\bar{3}}_2 \leftrightarrow \Omega^{0,2}(M_G)$ $\mathbf{1}_3 \leftrightarrow \Omega^{0,3}(M_G)$

Cohomology	Bosons	Fermions	Multiplet
$H^0(M_G)$	$v_\mu, \mu = 0,1$	$\eta_-, \bar{\eta}_-$	Vector
$H^1(M_G, L_R)$	a	ψ_+	Chiral (Wilson lines)
$H^2(M_G, L_R)$	$-$	$\bar{\rho}_-$	Conjugate-Fermi
$H^3(M_G, L_R)$	$\bar{\varphi}$	$\bar{\chi}_+$	Conjugate-chiral (deformations of M_G)

L_R: gauge background breaking Adj$(G) \to \bigoplus_R R$

Surface localised matter

- Two Kähler 3-cycles intersect over Kähler surface $S_{\mathbf{R}} = M_{G_1} \cap M_{G_2}$
- Extra massless matter in representations of $G_1 \times G_2$
- From perspective of M_{G_1}:
  $S_{\mathbf{R}}$ is a defect with trapped localised zero-modes
- In flat space: Defect theory is $N = (0,1)$ SYM on $\mathbb{R}^{1,5}$ with
  hypermultiplet in repr $\mathbf{R}$

Repeat twisting and deduce zero-modes in presence of gauge bundle $L_{\mathbf{R}}$:

Cohomology	States	Multiplet
$H^0_{\bar\partial}(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_{S_{\mathbf{R}}}})$	$(T, \tau_+)^{\mathbf{R}}$	chiral
$H^1_{\bar\partial}(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_{S_{\mathbf{R}}}})$	$(-, \bar\mu_-)^{\mathbf{R}}$	conjugate Fermi
$H^2_{\bar\partial}(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_{S_{\mathbf{R}}}})$	$(\bar S, \bar\sigma_+)^{\mathbf{R}}$	conjugate chiral

Kähler surface $S_{\mathbf{R}}$ not necessarily spin

Flux quantization condition $\Longrightarrow$ $L_{\mathbf{R}} \otimes \sqrt{K_{S_{\mathbf{R}}}}$ is honest bundle on $S_{\mathbf{R}}$

Cubic Interactions - I

Cubic interactions between bulk and surface matter fields

✓ descend from cubic gauge interaction of 8d SYM

✓ organize into holomorphic cubic couplings of 2d (0,2) theory

2 types of [Witten'93]

holomorphic interactions :

Multiplet	Field	Content
gauge	V	$(v_\mu; \eta_-, \bar{\eta}_-; \mathfrak{D})$
chiral	Φ	$(\varphi; \chi_+; -)$
Fermi	P	$(-; \rho_-; E, G)$

- E-type interaction: $E_a(\Phi_i)$

$$L^F = -\frac{1}{2}\int d^2y d^2\theta\, P_a \bar{P}_a \;= -\int d^2y \left(\bar{\rho}_{-,a}\frac{\partial E_a}{\partial \varphi_i}\chi_{+,i} + c.c.\right) + \dots$$

- Superpotential: $J^a(\Phi_i)$

$$L^J = -\frac{1}{\sqrt{2}}\int d^2y\, d\theta^+\; P_a J^a(\Phi_i)|_{\bar{\theta}^+=0} - \text{c.c.} = -\int d^2y \left(G_a J^a + \rho_{-,a}\frac{\partial J^a}{\partial \varphi_i}\chi_{+,i} + \text{c.c.}\right)$$

subject to constraint $\text{Tr} J^a(\Phi) E_a(\Phi) = 0$

Cubic Interactions - II

1) bulk $\times$ bulk $\times$ bulk

- Wilson line a – deformation mode φ:

  $E^{(\rho_-^\alpha)} = -\mathbf{f}_{\alpha\mu\epsilon}\,\Phi^\mu\,A^\epsilon$

  $S = \mathbf{f}_{\alpha\mu\epsilon}\int d^2y\;\bar{\rho}_-^\alpha\;\left(\varphi^\mu\,\psi_+^\epsilon + \chi_+^\mu\,a^\epsilon\right)\qquad \mathbf{f}_{\alpha\mu\epsilon} = \int_{M_G}\hat{\bar{\rho}}_{\bar{k}\bar{m},\alpha}\wedge\left(\hat{\varphi}_{kmn,\mu}\wedge\hat{\psi}_{\bar{n},\epsilon}\right)$

- Wilson line a – Wilson line a :

  $J_{(\rho_-^\alpha)} = -\mathbf{g}_{\alpha\beta\gamma}\,A^\beta\,A^\gamma$

  $S = \mathbf{g}_{\alpha\beta\gamma}\int d^2y\;\rho_-^\alpha\;a^\beta\;\psi_+^\gamma\qquad \mathbf{g}_{\alpha\beta\gamma} = \int_{M_G}\tilde{\rho}_{kmn\bar{n},\alpha}\wedge\hat{a}_{\bar{k},\beta}\wedge\hat{\psi}_{\bar{m},\gamma}$

2) $\text{bulk}|_{S_\mathbf{R}}$ $\times$ surface $\times$ surface

$$\begin{aligned}
S_{\text{bulk+matter}} &= S^{(F)}_{\text{bulk+matter}} + S^{(J)}_{\text{bulk+matter}}\\
S^{(F)}_{\text{bulk+matter}} &= \mathbf{b}_{\alpha\beta\gamma}\int d^2y\,\overline{\rho}_-^\alpha\left(\tau_+^\beta\,S^\gamma + \sigma_+^\gamma\,T^\beta\right) + \text{c.c.}\\
&\quad +\mathbf{e}_{\delta\gamma\epsilon}\int d^2y\,\overline{\mu}_-^\delta\left(S^\gamma\psi_+^\epsilon + \sigma_+^\gamma a^\epsilon\right) + \text{c.c.}\\
S^{(J)}_{\text{bulk+matter}} &= \mathbf{c}_{\delta\beta\epsilon}\int d^2y\,\mu_-^\delta\left(T^\beta\,\psi_+^\epsilon + \tau_+^\beta\,a^\epsilon\right) + \text{c.c.}
\end{aligned}$$

Cubic Interactions - III

Further enhancements from fibre splittings in codimension three and four:

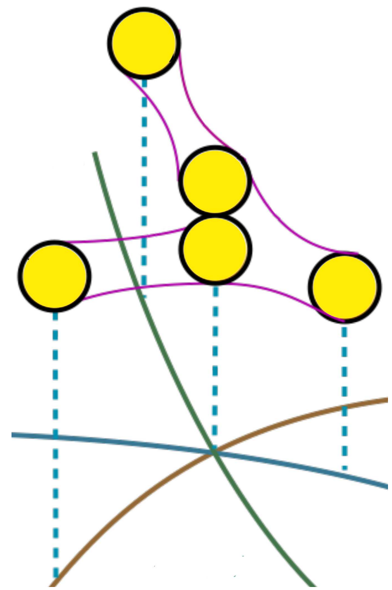
Codimension three curve

- $\Sigma_{\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3} = S_{\mathbf{R_1}} \cap S_{\mathbf{R_2}} \cap S_{\mathbf{R_3}}$
- fibre curve splitting:

$$C_{\lambda_{\mathbf{R}_1}} \to C_{\lambda_{\mathbf{R}_2}} + C_{\lambda_{\mathbf{R}_3}}$$

- gauge invariant contraction

$$\mathbf{R}_1 \oplus \mathbf{R}_2 \oplus \mathbf{R}_3 \;\to\; \mathbb{C}$$



cubic E and J-type couplings from wavefunction overlaps over $\Sigma_{\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3}$

[Schäfer-Nameki,TW'16] [Apruzzi,Hassler,Heckman,Melnikov'16]

$$J_{\left(\mu_-^{\mathbf{R}_1,\delta}\right)} = -\mathbf{h}_{\delta\epsilon\gamma}\left(\mathcal{Z}_2^{\mathbf{R}_2,\epsilon}\,\mathcal{Z}_3^{\mathbf{R}_3,\gamma}\right), \quad E^{\left(\mu_-^{\bar{\mathbf{R}}_2,\delta}\right)} = -\mathbf{d}_{\delta\epsilon\gamma}\left(\mathcal{Z}_1^{\mathbf{R}_1,\epsilon}\,\mathcal{Z}_3^{\mathbf{R}_3,\gamma}\right)$$

$\mathcal{Z} = \mathcal{S}$ or $\mathcal{T}$ - depending on which combination is gauge invariant

New: Quartic Interactions

Further enhancement points in base - codimension four

[Schäfer-Nameki,TW'16] [Apruzzi,Hassler,Heckman,Melnikov'16]

- At intersection of two coupling curves $\Sigma_{\mathbf{R}_i\mathbf{R}_j\mathbf{R}_j}$
- Structure of curve splitting in agreement with quartic couplings

$$\mathbf{R}_1 \oplus \mathbf{R}_2 \oplus \mathbf{R}_3 \ \oplus \mathbf{R}_4 \ \rightarrow \ \mathbb{C}$$

- In 2d scalar fields have mass dimension zero

  Couplings $(\text{fermion})^2 \times (\text{scalar})^2$ still 'super-renormalisable'
- Expect all combinations for J and E-type couplings in agreement with gauge invariance

CY5 embedding

Example: $G = SU(5)$ in CY 5-fold

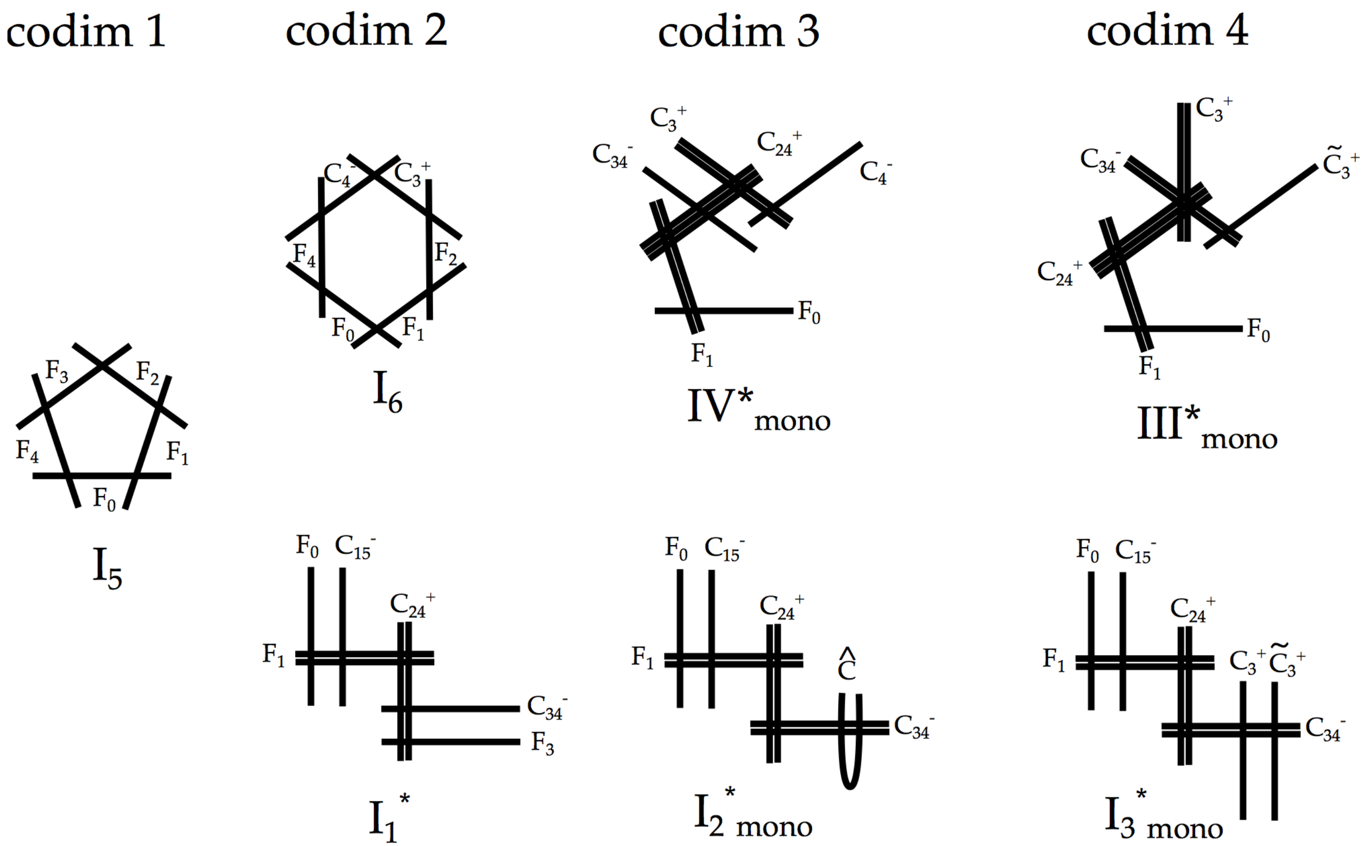
$$y^2 + b_1 xyz + b_3 w^2 yz^3 = x^3 + b_2 w x^2 z^2 + b_4 w^3 x z^4 + w^5 z^5 b_6 \,,$$

Discriminant at

$$\Delta = \quad w^5 \Big(b_1^4 \left(b_2 b_3^2 + b_1 b_3 b_4 + b_1^2 b_6 \right) + \mathcal{O}(w) \Big) \qquad w = 0 : \;\; SU(5) - \text{brane}$$

Codim 2 :	$SO(10)$:	$b_1 = 0$	**10** matter
	$SU(6)$:	$b_1^2 b_6 - b_1 b_3 b_4 + b_2 b_3^2 = 0$	**5** matter
Codim 3 :	$SO(12)$:	$b_1 = b_3 = 0$	$\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}$ coupling
	E_6 :	$b_1 = b_2 = 0$	$\mathbf{10}\,\mathbf{10}\,\mathbf{5}$ coupling
Codim 4 :	$SO(14)$:	$b_1 = b_3 = b_4^2 - 4 b_2 b_6 = 0$	$-$
	E_7 :	$b_1 = b_2 = b_3 = 0$	$(\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}} + \mathbf{10}\,\mathbf{5}\,\mathbf{5}\,\mathbf{5})$ couplings

CY5 embedding



Many more examples in [Schäfer-Nameki,TW'16]

- including with exceptional gauge algebra

D3-brane sector

D3-branes wrap curves C^B on base B_4

M-theory dual: M2-branes as dictated by tadpole cancellation condition

$$\delta([C^B_{\rm M2}]) = [\tfrac{1}{24}c_4(Y_5) - \tfrac{1}{2}G_4 \wedge G_4]|_B \in H^3(B_4)$$ cf. [Haupt,Lukas,Stelle'08]

Significance of D3/M2-sector:

- Intersection $C^B \cap M_G =$ points
- D3-7 strings 8 DN boundary conditions $\Rightarrow a_{NS} = -\frac{1}{2} + \frac{8}{8} = \frac{1}{2}$
- Only 1 fermionic degree of freedom

  $\Rightarrow$ Fermi multiplet ν_- in fundamental of 7-brane group G
- Number of Fermi multiplets ν_-: [Schäfer-Nameki,TW'16]

$$\frac{1}{\mathrm{ord}(g)} \times \int_{B_4} [M_G] \wedge [C^B_{\rm M2}]$$

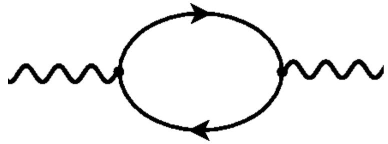
  g : extra monodromy group

Relevance: D3-7 states contribute to gauge anomalies!

Gauge anomalies

2d anomalies are quadratic:

$$\partial_\mu J^{\mu a} = \frac{1}{8\pi} \mathrm{Tr}(\gamma^3 T^a_{\mathbf{R}} T^b_{\mathbf{R}})\, F^b_{\mu\nu} \epsilon^{\mu\nu}$$



Anomaly coefficient

$$\mathcal{A}(\mathbf{R}, P) = P\, C(\mathbf{R}), \quad \mathrm{tr}\, T^a_{\mathbf{R}} T^b_{\mathbf{R}} = C(\mathbf{R})\, \delta^{ab}, \qquad P = \begin{cases} +1 & \text{chiral mult.} \\ -1 & \text{vector \& Fermi} \end{cases}$$

$$\mathcal{A}_{\text{total}} = \mathcal{A}_{\text{bulk}} + \mathcal{A}_{\text{surface}} + \mathcal{A}_{3-7}$$

- Total anomaly for non-abelian gauge group must vanish
- Guaranteed by anomaly inflow argument via D3-tadpole cancellation condition
- Has been checked explicitly in many examples with and without gauge flux [Schäfer-Nameki, TW'16]

Relation to GLSM

Have found 2d N=2 (0,2) gauge theories in decoupling limit

- contain non-abelian and/or abelian and/or discrete gauge groups
- Proper subset: 'vanilla' GLSMs with $U(1)$ only
  $\leftrightarrow$ recent progress in abelian gauge sector in F-theory

Crucial for GLSM interpretation:

Fayet-Iliopoulos term r_m for GLSM $U(1)_m$ $\longleftrightarrow$ Kahler parameter of heterotic target space

Origin of r_m in F-theory compactification:

- gauging of axions w.r.t. $U(1)_m$
- 2d Stückelberg mass for $U(1)_m$ in presence of flux G_4

2d (0,2) theories have rich set of Stückelberg and GS terms

[Mohri'96][Compean,Uranga'97] [Adams'09]; [Quigley,Sethi] [Blasczyk,Nibbelink,Ruehle]'11

Relation to GLSM

In F-theory on CY$_5$: [Schafer-Nameki,TW'16]

- $U(1)_m$ Fayet-Iliopoulos D-term $r_m = \int_{Y_5} G_4 \wedge S_m \wedge J_B \wedge J_B$

  S_m: rational section, J_B: Kähler form on B_4

- In suitable decoupling limit Kähler moduli of B_4 become non-dynamical and r_m becomes parameter in gauge theory

  in agreement with [Komargodski,Seiberg'10]

Different values of FI terms correspond to different 2d regimes:

[Schäfer-Nameki,TW'16]

$$\begin{array}{ccccc} \mathrm{NLSM-phase}\ r \gg 0 & & \mathrm{GLSM} & & \mathrm{LG-phase}\ r \ll 0 \\ G = \emptyset & \xleftarrow[\text{data}]{\text{gluing}} & G = U(1) & \xrightarrow[\text{data}]{\text{gluing}} & G = \mathbb{Z}_5 \\ (\tilde{A}, \tilde{\Phi}) & & (A, \Phi) & & (\hat{A}, \hat{\Phi}) \\ \mathrm{T-brane} & & & & \mathrm{T-brane} \end{array}$$

Summary

Codim	2d $N = (0,2)$ Gauge Theory
1	Gauge algebra + bulk matter $\mathfrak{g}$
	bulk couplings
2	Matter in $\mathbf{R} \oplus \bar{\mathbf{R}}$
	Bulk-surface couplings.: E and J
3	Matter couplings: E and J
4	Matter couplings: E and J

So far have studied 2d $N = (0,2)$ gauge theory via

- topological twist
- F/M-theory duality

Many new directions to explore, e.g.

- RG-flow to (0,2) SCFTs in the infra-red
  Geometrisation of RG flow by shrinking brane cycle volumes?
- Better non-perturbative understanding of D3-brane sector
- Full inclusion of 2d (0,2) SUGRA sector, grav. anomalies
- Worldsheet interpretation of 2d theory?
  non-abelian gauge theories — GLSM phases?