NILPOTENT SUPERGRAVITY AND APPLICATIONS

Based on :

- I.Antoniadis, E.D., S.Ferrara and A. Sagnotti, Phys.Lett.B733 (2014) 32 [arXiv:1403.3269 [hep-th]].
- E.D., S.Ferrara, A.Kehagias and A.Sagnotti, JHEP 1509 (2015) 217 [arXiv:1507.07842 [hep-th]].
- G. Dall'Agata, E.D., F.Farakos, JHEP 1605 (2016) 041, arXiv:1603.03416 [hep-th].

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Outline

1) Nonlinear SUSY realizations, constrained superfields

2) String realization

3) Nilpotent supergravity

4) Inflation with nilpotent superfields

5) Conclusions

1) Non-linear SUSY, constrained superfields

- In supergravity, the gravitino Ψ becomes massive by absorbing the goldstino G

$$\Psi_\mu \begin{pmatrix} 3/2 \\ - \\ - \\ -3/2 \end{pmatrix} + G \begin{pmatrix} - \\ 1/2 \\ -1/2 \\ - \end{pmatrix} = \Psi_\mu \begin{pmatrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$$

Goldstino is part of a multiplet $X = (x, G, F_X)$

One of the first written SUSY lagrangian had nonlinearly realized SUSY, Volkov-Akulov (VA), 1973.

$$\mathcal{L}_X = \int d^4\theta \, X^\dagger X + \left\{ \int d^2\theta \, f \, X + h.c. \right\}$$

$$= det \ (E_\mu^a) \ , \quad \text{where} \quad E_\mu^a = e_\mu^a + (\frac{i}{2f^2} G\sigma^a \partial_\mu \bar{G} + h.c.)$$

is the VA "vierbein". In the standard VA prescription, couplings to matter proceed as in gravity

$$G^{\mu\nu} \ T_{\mu\nu,M} \ = \ g^{\mu\nu} \ T_{\mu\nu,M} \ + \ (\frac{i}{2f^2} G\sigma^\mu \partial^\nu \bar{G} + h.c.) \ T_{\mu\nu,M}$$

- Constrained superfields

- VA action can be constructed in superspace (Rocek,78) introducing a constrained, nilpotent superfield

$$X^2 = 0$$

whose solution is no fundamental scalar

$$X = \frac{GG}{2F_X} + \sqrt{2}\,\theta\, G + \theta^2 F_X$$

The full VA action is

$$\mathcal{L}_{VA} = \left[X\overline{X} \right]_D + \left[f X + h.c. \right]_F$$

Analogy with the sigma model :

- O(N) linear sigma model

$$\mathcal{L} = \partial_m \phi_a \partial^m \phi_a - \lambda(\phi_a \phi_a - v^2)^2$$

has 1 massive (« Higgs ») and N-1 goldstone bosons,

versus the

- O(N)/ O(N-1) nonlinear sigma model ($\lambda \to \infty$ limit)

$$\mathcal{L} = \partial_m \phi_a \partial^m \phi_a$$

plus the constraint $\phi_a \phi_a = v^2$, which describes self-interactions of the N-1 goldstone's.

O(N) symmetry is nonlinearly realized.

There are two different cases to consider:

i) non-SUSY matter spectrum

$$E << m_{sparticles} \, , \, \sqrt{f}$$

⇨ non-linear SUSY in the matter sector

ii) SUSY matter multiplets : $(\tilde{q}, q)$, etc

$$m_{sparticles} \leq E << \sqrt{f}$$

⇨ linear SUSY in the matter sector

Case i) (non-linear matter) → additional constraints (KS)

- light fermions : X Q = 0 eliminates (complex) scalars

$$Q = \frac{1}{F_X}\left(\Psi_q - \frac{F_q G}{2F_X}\right)G + \sqrt{2}\theta\Psi_q + \theta^2 F_q$$

- light complex scalars : $X\bar{H} =$ chiral, eliminates fermions

$$H = h + i\sqrt{2}\theta\sigma^m\partial_m h\frac{\bar{G}}{\bar{F}_X} + \theta^2\left[-\partial_n\left(\frac{\bar{G}}{\bar{F}_X}\right)\bar{\sigma}^m\sigma^n\partial_m h\frac{\bar{G}}{\bar{F}_X} + \frac{1}{2\bar{F}_X^2}\bar{G}^2\partial^2 h\right]$$

In this case, there is no more auxiliary field.

- light real scalar (inflaton ?) : $X(\Phi - \bar{\Phi}) = 0$ eliminates a scalar (sinflaton ?), the fermion (inflatino ?) and the aux. field.

- Recent progress in the possible UV origin of superfield constraints (d'ADF; Kallosh, Karlsson,Mosk,Murli, 2016).
- Ex: decoupling a fermion with an UV operator

$$-\frac{m_\chi}{2f^2}\int d^4\theta\left[|X|^2 D^\alpha Y D_\alpha Y + c.c.\right]$$

which decouples only the fermion ($m_\chi \to \infty$) and produces the constraint

$$|X|^2 D_\alpha Y = 0, \qquad (*)$$

The KS constraint $\bar{X} D_\alpha Y = 0$ needs two operators

$$-\frac{m_h}{2f^2}\int d^4\theta\left[|X|^2 D^\alpha \mathcal{H} D_\alpha \mathcal{H} + c.c.\right] - \frac{g_{FH}}{f^2}\int d^4\theta\left[|X|^2 D^2 \mathcal{H} \overline{D}^2 \overline{\mathcal{H}}\right]$$

and eliminates also the auxiliary field, since it is equivalent to (*) plus

$$|X|^2 D^2 Y = 0$$

The inflaton constraint

$$X(\mathcal{A}-\overline{\mathcal{A}})=0.$$

can be analyzed similarly. It can be shown to be equivalent to

$$\begin{aligned} |X|^2(\mathcal{A}-\overline{\mathcal{A}}) &= 0, \\ |X|^2\overline{D}_{\dot\alpha}\overline{\mathcal{A}} &= 0, \\ |X|^2\overline{D}^2\overline{\mathcal{A}} &= 0. \end{aligned} \qquad (**)$$

The UV operators one needs are

$$\int d^4\theta\left[\frac{m_b^2}{2f^2}|X|^2(\mathcal{A}-\overline{\mathcal{A}})^2-\frac{g_{FA}}{f^2}|X|^2D^2\mathcal{A}\overline{D}^2\overline{\mathcal{A}}\right]-\frac{m_\zeta}{2f^2}\int d^4\theta\left[|X|^2D^\alpha\mathcal{A}D_\alpha\mathcal{A}+c.c.\right].$$

However, each individual constraint in (**) is independent of the others and can be imposed separately.

All superfield constraints can be written in terms of « primary constraints » of the type

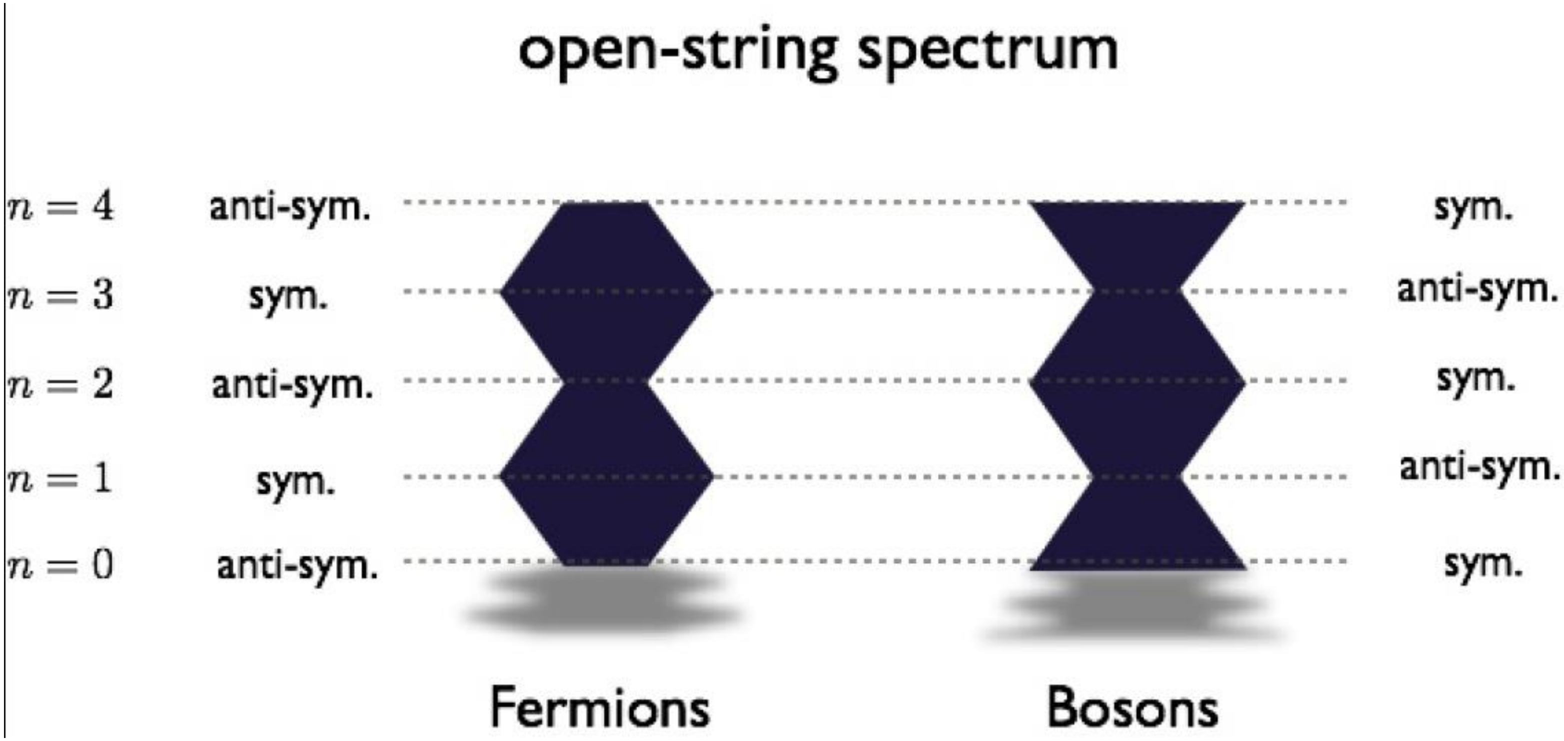
$$X \overline{X} Q_{\mathrm{L}} = 0$$

which decouples only one component (scalar,fermion or auxiliary field).

Obtaining the constraints starting from simple UV models more difficult (E.D., Heurtier,Wieck,Winkler,2016).

2) String realization

- The string theory realization of the nilpotent superfield is by the $\overline{D}3/O3$ system (Kallosh, Quevedo,Uranga, 2015). Explicit similar string vacua known (Brane SUSY breaking: (Sugimoto;Antoniadis,E.D.,Sagnotti, 1999; $\overline{D}p/Op_+$)



-SUSY is non-linearly realized on the antibranes (E.D.,Mourad, 2000).

-For one $\overline{D3}$ on top of an $O3_-$ the only degree of freedom is the goldstino.

-The $\overline{D3}$ action can be written with the goldstino nilpotent superfield (Kallosh,Wrase,2014; Bandos,Martucci,Sorokin,Tonin,2015.)

$\overline{D}3/O3$

also used in the KKLT moduli stabilization. The KKLT « uplift » can be written in a manifestly supersymmetric way in the nilpotent goldstino formalism (Kallosh-Linde,2014; Bergshoeff,Dasgupta,Kallosh, Van Proeyen,Wrase, 2015)

3) Nilpotent supergravities

In SUGRA, the most general couplings of the nilpotent X are described by (ADFS,2014)

$$K = -3\log\left(1 - X\overline{X}\right) \equiv 3\,X\,\overline{X}\ , \qquad W = f\,X + W_0$$

and as a result

$$\mathcal{L}_{mass} = -\,m_{3/2}\left(\psi_m + \frac{i}{\sqrt{6}}\,\sigma_m\,\overline{G}\right)\sigma^{mn}\left(\psi_n + \frac{i}{\sqrt{6}}\,\sigma_n\,\overline{G}\right) + \text{h.c.}$$

The SUGRA lagrangian contains the proper goldstino couplings and

$$V = \frac{1}{3}\,|f|^2 - 3\,|W_0|^2\ , \qquad m_{3/2}^2 = |W_0|^2$$

Recently the complete lagrangian was written by
Bergshoeff, Freedman,Kallosh, Van Proeyen and Hasegawa,Yamada (2015)

Consider the gravity multiplet,

$$(e_m^a, \psi_m^\alpha, u, A_m)$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

vierbein gravitino auxiliary fields

coupled to the goldstino multiplet X, in the decoupling limit. The theory contains actually just
the graviton + one massive gravitino

There should be a purely gravitational description, with a modified/constrained gravity multiplet.

- Nilpotent supergravity (DFKS ; Antoniadis,Markou, 2015)

- This is described by

$$\mathcal{L} = \left[- S_0 \, \overline{S}_0 \right]_D + \left[W_0 S_0^3 \right]_F$$

with the constraint $\left(\frac{\mathcal{R}}{S_0} - \lambda \right)^2 = 0$, where :

- $\mathcal{R}$ = chiral curvature
- S_0 = chiral compensator field
- λ is related to the cosmological constant ⟹

$$\overline{u} - \lambda = \frac{2\, i \, \widehat{\mathcal{D}_\mu \overline{\psi}_\nu}_L \, \gamma^{\mu\nu} \gamma^{\rho\sigma} \, \widehat{\mathcal{D}_\rho \psi_\sigma}_L}{\alpha + \frac{4\,\lambda^2}{3}} \left(1 - \frac{8\, i\, \lambda}{3} \, \frac{\widehat{\mathcal{D}_\mu \overline{\psi}_\nu}_R \, \gamma^{\mu\nu} \gamma^{\rho\sigma} \, \widehat{\mathcal{D}_\rho \psi_\sigma}_R}{\left| \alpha + \frac{4\,\lambda^2}{3} \right|^2} \right)$$

where

$$\alpha = - \widehat{R} + \frac{2}{3} A_\mu^2 - 2\, i \, \widehat{\mathcal{D}^\mu A_\mu} ,$$

whereas the goldstino is

$$G \; = \; - \, \frac{3}{2\,\lambda} \left(\gamma^{\mu\nu} \, \partial_\mu \, \psi_\nu \; - \; \frac{\lambda}{2} \, \gamma^\mu \, \psi_\mu \right)$$

This describes just the gravitational multiplet, with nonlinear SUSY.

We show this is exactly dual to the Volkov-Akulov SUGRA :
Introduce two lagrange multipliers X, Λ_1 that « linearize » the lagrangian

$$\mathcal{L} \; = \; \left[- \; S_0 \, \overline{S}_0 \right]_D \; + \; \left[\left\{ X \left(\lambda \; - \; \frac{\mathcal{R}}{S_0} \right) \; - \; \frac{1}{4\Lambda_1} \, X^2 \; + \; W_0 \right\} S_0^3 \right]_F$$

Use the identity

$$\Big[f(\Lambda)\, \mathcal{R}\, S_0^2 \Big]_F \ + \ \text{h.c.} \ = \ \Big[\big(f(\Lambda) \ + \ \overline{f}(\overline{\Lambda}) \big)\, S_0\, \overline{S}_0 \Big]_D \ + \ \text{tot. deriv.}$$

which leads to

$$\mathcal{L} \ = \ \Big[\ - \ (1 \ + \ X \ + \ \bar{X}) \ \ S_0\, \overline{S}_0 \Big]_D \ + \ \Big[\Big\{ \lambda\, X \ - \ \frac{1}{4\Lambda_1} \ X^2 \ + \ W_0 \Big\}\, S_0^3 \Big]_F$$

One finally finds a standard SUGRA with

$$K \ = \ -\, 3 \, \ln \big(1 + X \ + \ \overline{X} \big) \ , \qquad W \ = \ W_0 \ + \ \lambda\, X$$

plus the constraint $X^2 = 0$, or equivalently

$$K \ = \ 3\, |X|^2 \ , \qquad W \ = \ W_0 \ + \ (\lambda \ - \ 3\, W_0) X$$

Coupling to matter (chiral multiplets Q) in nilpotent SUGRA can be implemented starting from

$$\mathcal{L} = \left[- e^{-\frac{1}{3} K_0(Q_i,\bar{Q}_{\bar{i}})} \, S_0 \, \overline{S}_0 \right]_D + \left[W_0(Q_i) \, S_0^3 \right]_F$$

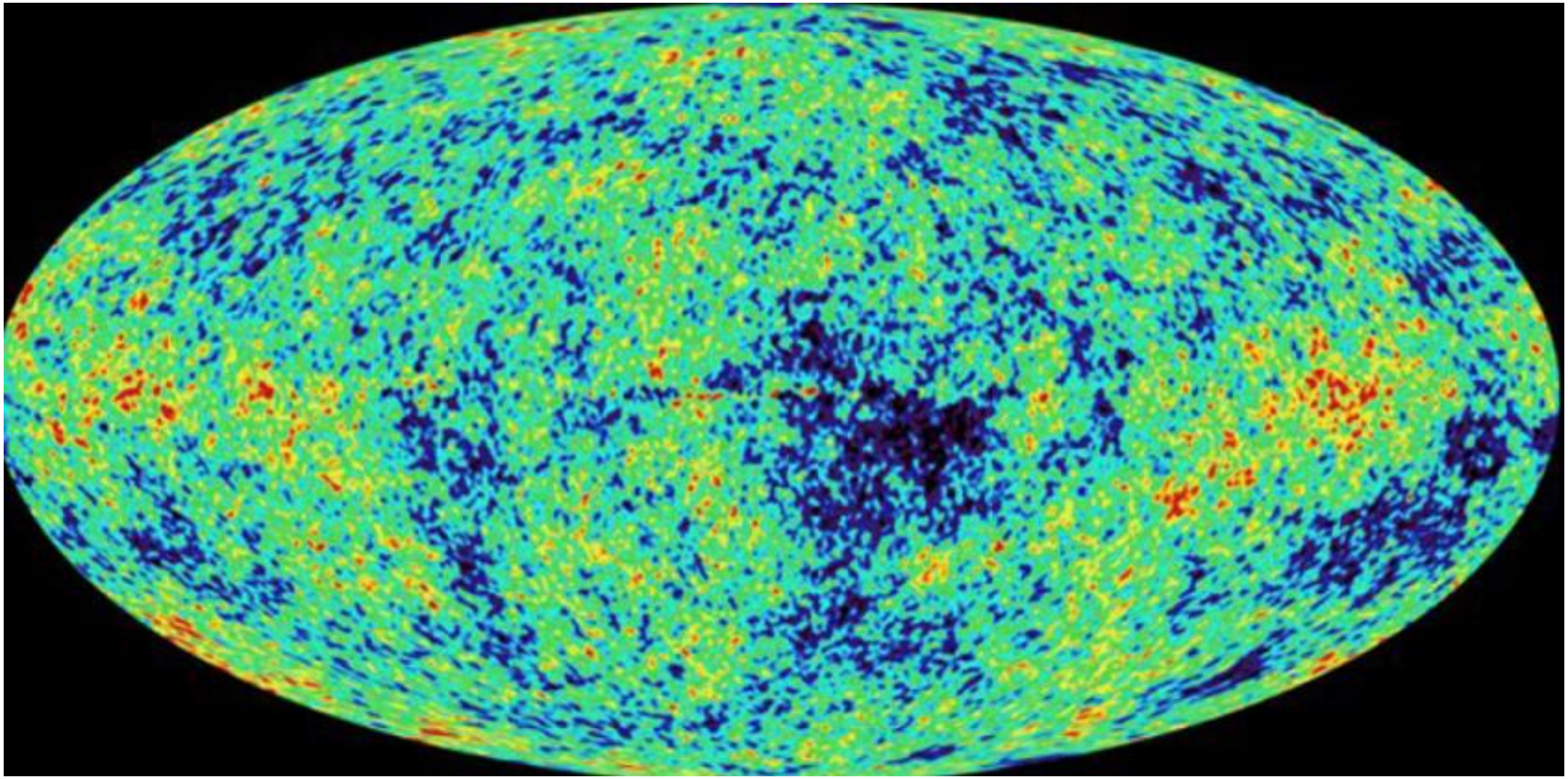
supplemented by the nilpotency constraint

$$\left(\frac{\mathcal{R}}{S_0} - f(Q_i) \right)^2 = 0$$

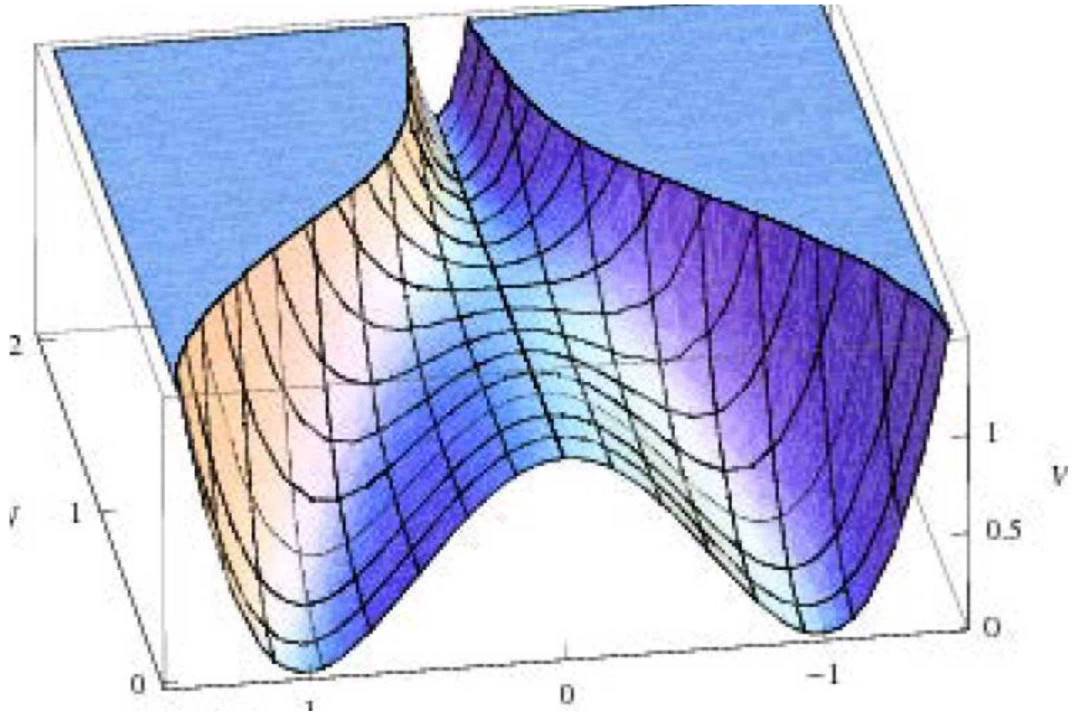
It can be shown that there is a consistency condition on the couplings

$$K_{0,\bar{i}} \, K_0^{\bar{i}j} \, K_{0,j} < 3$$

4) Inflation with nilpotent superfields



Cosmological data favors models with one light inflation



Simplest SUGRA models contain 2 complex scalars for inflation and 3 complex scalars if SUSY breaking. Desirable to have simpler models.

Use nilpotent constraints to eliminate fields (ADFS,2014; Kallosh-Linde; talks Kallosh,Kehagias,Quevedo,Wrase).

Simplest models: inflaton multiplet + X , $X^2 = 0$

There are consistency conditions to be satisfied (Dall'Agata-Zwirner,2014) :

- $F_X \neq 0$ during inflation and afterwards
- unitarity. In flat background, unitarity valid for

$$E^2 < m_{3/2} M_P$$

During inflation, this should be replaced by

$$E_q^2 < m_{3/2}(\varphi) M_P$$

where $E_q \sim V_{\rm inf}^{1/2}(\varphi)/M_P$ is the typical energy scale of quantum fluctuations during inflation.

Several inflationary models constructed afterwards

((Ferrara) Kallosh,Linde, Dall'Agata-Zwirner… 2014-2016)

Simple models constructed based on

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + |X|^2 \quad , \quad W = f(\Phi)(1 + \sqrt{3}X)$$

with $f(0) \neq 0, f'(0) = 0$ and $\overline{f(x)} = f(-\bar{x})$

The inflationary potential is $V = \left| f'\left(i \frac{\varphi}{\sqrt{2}} \right) \right|^2$

Inflationary energy is decoupled from the gravitino mass: possible to have GUT vacuum energy with low (TeV) gravitino mass.

Explicit examples :

$$f(\Phi) = f_0 - \frac{m}{2}\Phi^2$$ (chaotic inflation)

$$f(\Phi) = f_0 - i\sqrt{V_0}(\Phi + i\frac{\sqrt{3}}{2}e^{\frac{2i\Phi}{3}})$$ (Starobinsky)

Perturbative unitarity OK if

$$|f'(i\frac{\varphi}{\sqrt{2}})| < |f(i\frac{\varphi}{\sqrt{2}})|$$

This is satisfied in both examples and is insensitive on the scale of SUSY breaking in the vacuum.

In nilpotent SUGRA, coupling to matter can lead to positive definite potentials and inflation (DFKS)

$$K_0 \; = \; - \, 3 \; \ln \left(1 \; - \; \frac{1}{2} \, (\Phi + \bar{\Phi})^2 \; - \; |Q|^2 \right)$$

$$W \; = \; W_0(\Phi, Q_i) \; + \; f(\Phi, Q_i) \, X$$

where

$$W_0 \; = \; \alpha(\Phi) \; + \; \frac{1}{2} \, b_{ij} \, Q_i \, Q_j \; + \; \frac{1}{6} \, \lambda_{ijk} \, Q_i \, Q_j \, Q_k$$

$$f \; = \; 6 \, \alpha(\Phi) \; + \; b_{ij} \, Q_i \, Q_j \; ,$$

The scalar potential is a generalization of Dall'Agata-Zwirner

$$V \; = \; \frac{1}{3Y^2} \left\{ \sum_i \left| \frac{\partial W_0}{\partial Q_i} \right|^2 + \left| \frac{\partial W_0}{\partial \Phi} \right|^2 + \frac{1}{2} \, (\Phi + \bar{\Phi})^2 \, |f|^2 + (\Phi + \bar{\Phi}) \left(\bar{f} \, \frac{\partial W_0}{\partial \Phi} \; + \; \text{h.c} \right) \right\}$$

Conclusions, Perspectives

- Volkov-Akulov SUGRA has a description with **only the gravity multiplet** ⟹ **nilpotent SUGRA.** **Constraints** on couplings to matter worked out . Component action ?

- Progress in the UV origin of constrained superfields. Several constraints possible to eliminate fermions, different effective actions.

- Using constrained superfields one can now construct simpler models (for ex. only the inflaton + gravity multiplet); SUSY automatically broken.

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- Consistency conditions to be satisfied in nilpotent SUGRA during inflation. Are they sufficient ?
- Minimal framework to include moduli stabilization, reheating.

Thank you

Backup slides

Large literature on SUSY non-linear realizations and low-energy goldstino interactions

- Volkov-Akulov, Ivanov-Kapustnikov
- - Siegel
- Casalbuoni, Dominicis, de Curtis, Feruglio, Gatto
- Brignole, Feruglio, Zwirner; Brignole
- Komargodski and Seiberg (KS)...