Type II model building at the crossroads between (astro)particle physics and moduli stabilisation

Gabriele Honecker

Institut für Physik, JG|U Mainz

based on various collaborations, in particular with
Mikel Berasaluce-González, Michael Blaszczyk, Jill Ecker, Isabel Koltermann, **Alexander Seifert, Wieland Staessens**

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String Pheno 2016, Ioannina, 23 June 2016

Motivation: String Theory - Anything / Nothing Goes??

Current Status:

- **particle physics**
  - several landscapes of (B)SM vacua (✓)

    see e.g. talks by Nilles, Faraggi, Ovrut, Palti, Cvetič, Lukas, Abel
  - typically SUSY @ M_{string}
    - controlled ~~SUSY~~: gaugino condensate
    - ...? Fluxes? Polonyi term?
    - non-SUSY possible if $M_{\text{string}} \sim \mathcal{O}(\text{TeV})$
  - ...excess of vector-like states ... **BSM signatures** at colliders?

    see e.g. talks by Antoniadis, Lüst
- **cosmology**
  - dS vacua?
  - complete **moduli** stabilisation?
  - inflationary potentials e.g. with **axions**?

  see e.g. plenary talks by Shiu, Blumenhagen, Partouche, Kallosh, Kiritsis, Cicoli, Quevedo, Marchesano, Staessens, Ibáñez, Hebecker, Reece + many parallel talks
- **mutual consistency??**

$\rightsquigarrow$ need to **reunite** (astro)particle & cosmological considerations

Motivation cont'd: Anything / Nothing Goes??

Old paradigm in Type II:

- gravity on closed strings
- gauge sector on D-branes

$\rightsquigarrow$ cosmo & particle physics decoupled

But:

- 4D (matter) field theory depends on dilaton & moduli
- moduli stabilisation changes geometry $\rightsquigarrow$ D-branes affected
- *'(anti) D-brane uplifting'* used to generate dS vacua
- ~~SUSY~~ mediation (hidden $\rightarrow$ observable sector) via gravity
- open & closed (pseudo)scalars mix, ν's, $U(1)$'s ...

$\rightsquigarrow$ *see later*

Motivation cont'd: Technical Challenges & Misconceptions

Technically challenging:

- geometric understanding with fluxes
- control over field theory
  for general (CY_3) backgrounds (without/with fluxes)

Vast misconception:

- large string landscape does not lead to arbitrary new physics

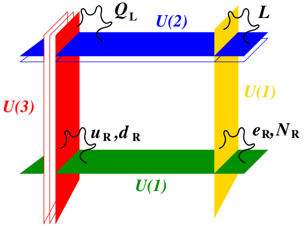
BSM Features: Particle Spectrum

BSM Features from Type II String Theory

Group theory on D-branes:

- charges at endpoints of open strings: $(\mathbf{N}_a, \overline{\mathbf{N}}_b)$, $(\mathbf{Adj}_a)$ of $U(N_a) \times U(N_b)$ $(\mathbf{N}_a, \mathbf{N}_b)$, $(\mathbf{Anti}_a)$, $(\mathbf{Sym}_a)$
- $U(1)_{\text{massless/massive}}$ linear combinations
  - $SU(5)$ GUTs
  - $SU(4) \times SU(2)_L \times SU(2)_R$ ✓
  - $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ ✓
  - $SU(3) \times SU(2) \times U(1)_Y$ ✓

Cvetič *et al.*; **G.H.** with Ott, Gmeiner, Ripka, Staessens, Ecker; ...



- $U(N) \to SO(2N)$: no spinor reps.

$\rightsquigarrow$ very limited choice of gauge groups & reps.

Hidden sectors & SM exotics:

- typically small rank, e.g. at most $SU(4)$
  - critical for ~~SUSY~~ via gaugino condensation
- since $\prod_a U(N_a) \subset SO(32)$: hard to hide G_{hidden} completely
  - vector-like exotics can acquire masses

Intermezzo: Comparison with Heterotic & F-Theory

Heterotic $SO(32)$

- S-dual to Type I

Heterotic $E_8 \times E_8$ **& F-Theory**

- *slight* enhancement of representations $\subset (\mathbf{Adj}_{E_8})$
- completely *hidden* sector ✓
- het.: geometric intuition?
- F-theory: control over field theory for g_{string} large??

Type IIA Example

Example: D6-Branes on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R})$

Why IIA string theory on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R})$?

- on **orbifolds**:
  - SUSY (D6-branes) = *sLag* condition ✓
  - CFT methods for spectrum ✓ & interactions (✓)

$$\theta^m \omega^n : z^k \to e^{2\pi i m v_k + n w_k} z^k \begin{cases} \mathbb{Z}_2 : & \vec{v} = \frac{1}{2}(1,-1,0) \\ \mathbb{Z}_6 : & \vec{w} = \frac{1}{6}(0,1,-1) \end{cases} \qquad \mathcal{R} : z^k \to \overline{z}^k$$

- discrete torsion possible due to $\mathbb{Z}_2 \times \mathbb{Z}_2$ subgroup:

$$\begin{pmatrix} h_{11} \\ h_{21} \end{pmatrix} = \begin{pmatrix} 3 + 2 \times 4_{\mathbb{Z}_6} + 8_{\mathbb{Z}_3} \\ 1_{\text{bulk}} + 2_{\mathbb{Z}_6} + 2_{\mathbb{Z}_3} + (6 + 2 \times 4)_{\mathbb{Z}_2} \end{pmatrix} = \begin{pmatrix} 19 \\ 19 \end{pmatrix}$$

- many **3-cycles** to wrap D6-branes

Ex: MSSM on rigid D6-branes on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R})$

Ecker, **G.H.**, Staessens '15

- **MSSM matter** of $\boxed{(SU(3)_a \times USp(2)_b \times SU(4)_h)^{(U(1)_{PQ}),\mathbb{Z}_3}_{U(1)_Y}}$:

$$3 \times \Big[(\mathbf{3},\mathbf{2},\mathbf{1})^{(0),0}_{1/6} + \underbrace{2 \times (\overline{\mathbf{3}},\mathbf{1},\mathbf{1})^{(1),1}_{1/3} + (\mathbf{3},\mathbf{1},\mathbf{1})^{(1),1}_{-1/3}} + (\overline{\mathbf{3}},\mathbf{1},\mathbf{1})^{(1),1}_{-2/3}\Big]$$

$$+ 3 \times \Big[\underbrace{(\mathbf{1},\mathbf{2},\mathbf{1})^{(1),1}_{1/2} + 2 \times (\mathbf{1},\mathbf{2},\mathbf{1})^{(1),1}_{-1/2}} + (\mathbf{1},\mathbf{1},\mathbf{1})^{(-2),1}_{0} + (\mathbf{1},\mathbf{1},\mathbf{1})^{(0),0}_{1}\Big]$$

$$= 3 \times \Big[Q_L + \overbrace{2 \times d_R + \overline{d_R}} + u_R\Big] + 3 \times \Big[\overbrace{H_u/\overline{L} + 2 \times L} + \nu_R + e_R\Big]$$

- **Higgses**: $3 \times \Big[(\mathbf{1},\mathbf{2},\mathbf{1})^{(1),1}_{1/2} + h.c.\Big] + \tilde{2} \times \Big[(\mathbf{1},\mathbf{2},\mathbf{1})^{(-1),2}_{1/2} + h.c.\Big]$
- **axions:** $3 \times [\Sigma^{cd} + \tilde{\Sigma}^{cd}] = \mathbf{3} \times \Big[(\mathbf{1},\mathbf{1},\mathbf{1})^{(-2),1}_{\mathbf{0}} + \mathbf{h.c.}\Big]$
- **SM vector-like states:** $(5_{\mathbf{Anti}_b} + 4_{\mathbf{Adj}_c} + 5_{\mathbf{Adj}_d}) \times (\mathbf{1},\mathbf{1},\mathbf{1})^{(0),0}_{0} +$

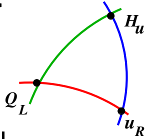
$$+ \Big[2 \times (\mathbf{3},\mathbf{1},\overline{\mathbf{4}})^{(0),1}_{1/6} + (\mathbf{3},\mathbf{1},\mathbf{4})^{(0),2}_{1/6} + 2 \times (\mathbf{3_A},\mathbf{1},\mathbf{1})^{(0),0}_{1/3} + h.c.\Big]$$

$$+ \Big[3 \times (\mathbf{1},\mathbf{1},\mathbf{1})^{(0),0}_{1} + (\mathbf{1},\mathbf{1},\mathbf{1})^{(-2),1}_{1} + 2 \times (\mathbf{1},\mathbf{1},\mathbf{6_A})^{(0),1}_{0} + h.c.\Big]$$

$$+ 3 \times (\mathbf{1},\mathbf{2},\mathbf{4})^{(0),2}_{0} + 6 \times (\mathbf{1},\mathbf{1},\overline{\mathbf{4}})^{(-1),0}_{-1/2} + 3 \times (\mathbf{1},\mathbf{1},\overline{\mathbf{4}})^{(-1),0}_{1/2} + 3 \times (\mathbf{1},\mathbf{1},\mathbf{4})^{(-1),1}_{1/2}$$

Yukawa Couplings and Masses for Exotics

Matter couplings from D-brane intersections:



- e.g.: $y_u^{(ijk)}\, Q_L^{(i)} \cdot \tilde{H}_u^{(j)} u_R^{(k)}$

off-diagonal	diagonal
$y_u^{(221)} \sim \mathcal{O}\left(e^{-\frac{16v_2+v_3}{48}}\right)$	$y_u^{(121)} \sim \mathcal{O}\left(e^{-\frac{4v_2+v_3}{48}}\right)$
$y_u^{(312)} \sim \mathcal{O}\left(e^{-\frac{v_2+16v_3}{48}}\right)$	$y_u^{(313)} \sim \mathcal{O}\left(e^{-\frac{v_2+4v_3}{48}}\right)$

v_i: volume of $T^2_{(i)}$

- 3^{rd} generation heavier if $v_3 < v_2$
- diagonal terms dominant if $v_2 < 5v_3$

$\rightsquigarrow$ *mild* **anisotropy** phenomenologically favoured

Here: free choice of parameters fits to pheno. acceptable scales
$\rightsquigarrow$ fits for **gauge couplings** along same lines

Challenge: large masses from e.g. $\overline{d}_R^{(3)} \Sigma^{cd(2)} d_R^{(6)} \sim \mathcal{O}\left(e^{-\frac{v_2+v_3}{12}}\right)$

Gauge Couplings @ 1-Loop and the Value of M_{string}

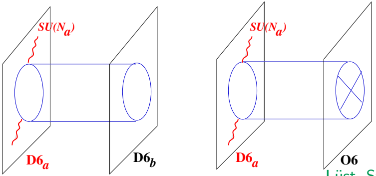
Gauge couplings $\leftrightarrow$ tuning of volumes:

- tree-level **gauge coupling**

$$\mu_6 \int_\Pi e^{-\Phi}\, \mathrm{tr} F_{\mu\nu} F^{\mu\nu} \to \boxed{\frac{\mathrm{Vol}_3}{4\, g_{\text{string}}}} \int_{4D} \mathrm{tr} F_{\mu\nu} F^{\mu\nu}$$

on $(T^2)^3/\Gamma$: $\mathrm{Vol}_3 = \sqrt{v_1 v_2 v_3}$

- **1-loop correction** via **CFT** - works on orbifolds only:



$$\sim \frac{b_a}{2} \ln \frac{M^2_{\text{string}}}{\mu^2} + \frac{\Delta_a}{2}$$

Lüst, Stieberger '03; Blumenhagen *et al.* '07; Gmeiner, **G.H.** '09

- from Δ_a:

v_i: Kähler modulus of $T^2_{(i)}$

$$\Re\left(\delta_b^{\text{1-loop}} \mathrm{f}_{SU(N_a)}\right) \supset -\frac{\tilde{b}^{\mathcal{A},(i)}_{ab}}{4\pi^2} \ln\left(e^{-\frac{\pi(\sigma^i_{ab})^2 v_i}{4}} \frac{\vartheta_1\left(\frac{\tau^i_{ab} - i\sigma^i_{ab} v_i}{2}, i v_i\right)}{\eta(i v_i)}\right) \overset{v_i \to \infty}{\sim} \pm v_i$$

- $M_{\text{string}} \ll M_{\text{GUT}}$: **tree $\leftrightarrow$ 1-loop cancellation**

$\rightsquigarrow$ new avenues for string phenomenology **G.H.**, Ripka, Staessens '12 ...

Example of Fine Tuning in Gauge Couplings @ 1-Loop

G.H., Koltermann, Staessens: in preparation

MSSM example on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R})$

- $\text{Vol}(\Pi_{a,h}) = R_1^{(1)} r^{(2)} r^{(3)} = \frac{1}{3}\text{Vol}(\Pi_{b,c,d})$ with $\frac{1}{g^2_{a,\text{tree}}} \propto \frac{\text{Vol}(\Pi_a)}{k_a}$

  $k_a = 1, 2$ for $SU(N_a)$, $USp(2N_a)$

$$\boxed{\frac{1}{g^2_{SU(3)_a}} = \frac{1}{g^2_{SU(4)_h}} = \frac{2}{3}\frac{1}{g^2_{USp(2)_b}} = \frac{6}{19}\frac{1}{g^2_{U(1)_Y}} \quad \text{@ tree level}}$$

- **1-loop** corrections: **annulus** contributions **only**

$$\delta^{\text{loop}}_{\mathcal{A}} \frac{1}{g^2_{SU(3)_a/SU(4)_h}} \stackrel{v_i\to\infty}{\longrightarrow} \frac{v_1}{4} \pm \frac{v_2}{24}$$

$$\delta^{\text{loop}}_{\mathcal{A}} \frac{1}{g^2_{USp(2)_b}} \stackrel{v_i\to\infty}{\longrightarrow} \frac{7\,v_1}{12} - \frac{v_2}{24} - \frac{v_3}{16}$$

$$\delta^{\text{loop}}_{\mathcal{A}} \frac{1}{g^2_{U(1)_Y}} \stackrel{v_i\to\infty}{\longrightarrow} \frac{25\,v_1}{18} + \frac{7\,v_2}{72}$$

- degeneracy lifted: $\frac{1}{g^2_{SU(3)_a,\text{loop}}} > \frac{1}{g^2_{SU(4)_h,\text{loop}}}$
- if $v_2 \gg v_1$: $\frac{1}{g^2_{G,\text{loop}}} < \frac{1}{g^2_{G,\text{tree}}}$ for $G = SU(4)_h, USp(2)_b$

Generic BSM Features: Scales & Field Theory

String Particle Physics and Scales

General insight from Type II:

- $M_{\text{string}} \ll M_{\text{GUT}}$ possible
  - depends on 1-loop cancellations for gauge couplings
- unisotropic compact directions often pheno. preferred
- fine tuning of SM singlet *vevs* for Yukawas & $\text{masses}_{\text{vector-like}}$
  $\rightsquigarrow$ *further complicated by interplay with cosmological constraints*
- discrete $\mathbb{Z}_n \subset U(1)_{\text{massive}}$ symmetries
  Berasaluce-Gonzalez, Ibáñez, Soler, Uranga '11; **G.H.**, Staessens '13
- new particles severely constrained by $\cancel{(\mathbf{N}_a, \mathbf{N}_b, \mathbf{N}_c)}$, $\cancel{(\mathbf{N}_a, \mathbf{Adj}_b)}$, $\cancel{(\mathbf{N}_a, \mathbf{Anti}_b)}$ ...

So far:

- all closed string moduli (& dilaton) as free parameters

Moduli Stabilisation at the orbifold point

Twisted Moduli Stabilisation on Orbifolds

Blaszczyk, **G.H.**, Koltermann '14-15

- IIA language: **complex structures** couple to D6-branes
  - **stabilisation** by **D-terms** of $U(1)$ factors $\quad (\Leftrightarrow \int_{\Pi_a} \mathrm{Im}(\Omega_3) \neq 0)$

$$\Pi_a^{\rm frac} = \frac{1}{4}\Pi_a^{\rm bulk} + \frac{1}{4}\sum_{i=1}^{3}\Pi_a^{\mathbb{Z}_2^{(i)}} = \Pi_{a,\mathrm{even}}^{\rm frac} + \Pi_{a,\mathrm{odd}}^{\rm frac}$$

$$= \frac{1}{4}\left(P_a\rho_1 + Q_a\rho_2 + U_a\rho_3 + V_a\rho_4\right) + \frac{1}{4}\sum_{\alpha=0}^{5}\left(x_{\alpha,a}^{(1)}\,\varepsilon_\alpha^{(1)} + y_{\alpha,a}^{(1)}\,\tilde{\varepsilon}_\alpha^{(1)}\right) + \frac{1}{4}\sum_{l=2,3}\sum_{\alpha=1}^{4}\left(x_{\alpha,a}^{(l)}\,\varepsilon_\alpha^{(l)} + y_{\alpha,a}^{(l)}\,\tilde{\varepsilon}_\alpha^{(l)}\right),$$

- modulus stabilised if $\Pi_{a,\mathrm{odd}}^{\rm frac} \neq 0$ contribution $(\leqslant h_{21}^{\mathbb{1}+\mathbb{Z}_2})$
- flat direction otherwise
  - either: gauge couplings change
  - or: particle physics insensitive

models from Ecker, **G.H.**, Staessens '14-15

Counting of stabilised complex structure moduli & flat directions with $1/g^2_{\mathbf{D6}_{x\in\{a,b,c,d,h_{1,2}\}}}$ dependence

$\mathbb{Z}_2\times\mathbb{Z}_6$	ϱ	$\varepsilon^{(1)}_{0,1,2}$	$\varepsilon^{(1)}_{3}$	$\varepsilon^{(1)}_{4+5}$	$\varepsilon^{(1)}_{4-5}$	$\varepsilon^{(2)}_{1,2}$	$\varepsilon^{(2)}_{3,4}$	$\varepsilon^{(3)}_{1,2}$	$\varepsilon^{(3)}_{3,4}$	$\#_{\rm stab}$
MSSM	none		a, c, h		$[b, d]_{\rm flat}$	c, d	none	a, d, h	none	6
PS I	none		a, h		$[b, c]_{\rm flat}$	$[a, b, c, h]_{\rm flat}$	none	a, h	none	4
PS II	none	h	a, h	a	$[b, c]_{\rm flat}$	$[a, b, c, h]_{\rm flat}$	none	a, h	none	7
L-R I	none	$h_{1,2}$	$a, d, h_{1,2}$	a, d	$[b, c]_{\rm flat}$	$h_{1,2}$	none	a, d	none	9
L-R II	none	$h_{1,2}$	$a, d, h_{1,2}$	a, d	$[b, c]_{\rm flat}$	$[a, b, c, d, h_{1,2}]_{\rm flat}$	none	$a, d, h_{1,2}$	none	7
L-R IIb	none	$h_{1,2}$	$a, d, h_{1,2}$	a, d	$[b, c]_{\rm flat}$	$[a, b, c, d]_{\rm flat}$	$[h_{1,2}]_{\rm flat}$	a, d	$h_{1,2}$	9
L-R IIc	none		a, d	a, d	$[b, c, h_{1,2}]_{\rm flat}$	h_1	h_2	a, d, h_1	h_2	10

Twisted Moduli Stabilisation on Orbifolds cont'd

- SM example on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_6 \times \Omega\mathcal{R})$:
  - 6 c.s. expected to be stabilised
  - $8 = 1_{\text{bulk}} + 7_{\mathbb{Z}_2}$ c.s. flat & decoupled
  - $1_{\mathbb{Z}_2}$ c.s. flat & coupling to $USp(2)_b \times U(1)_Y$
    ($\frac{1}{g_x^2} \propto \text{Vol}_x \propto \text{Vol}_x^0 \mp \sqrt{\varepsilon_{4-5}^{(1)}}$ behaviour expected)
    $\Leftrightarrow$ interplay with 1-loop corrections to gauge couplings

G.H., Koltermann, Staessens in progress

- twisted **Kähler** moduli **decoupled** from particle phyics

Further stabilisation by fluxes:

- backreaction on geometry
- fluxes & chirality per sector exclude each other ($\lightning$)

Towards Moduli Stabilisation by Non-Factorisable Tori

Towards Model Building on Non-Factorisable Backgrounds

Berasaluce-González, **G.H.**, Seifert '16, cf. Seifert's talk on Tuesday

Motivation:

- 3-form flux on $T^3 \times \ldots$ geometry
- *a priori* less closed string moduli

cf. CFT methods "local" RR tapdole cancellation: Blumenhagen, Conlon, Suruliz '04

Example: $T^6/(\mathbb{Z}_4 \times \Omega\mathcal{R})$ **with** $\vec{v} = \frac{1}{4}(1, -2, 1)$

- possible: $(T^2)^3$ or $(T^3)^2$ or $\boxed{T^3 \times T^1 \times T^2}$
  - 3-cycle geometry rich enough for model building
  - (pairwise) symmetries between lattice orientations under $\Omega\mathcal{R}$
    - twice as many backgrounds /Blumenhagen *et al.*/: $\mathbf{A_aAA}$ and $\mathbf{A_bAA}$
    - geometric method for generic values of moduli
  - **new:** fractional *Lag*s can be rewritten on $(T^2)^3$
    $\rightsquigarrow$ CFT methods accessible for spectrum & interactions

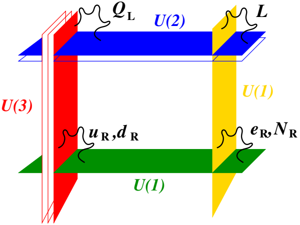
Berasaluce-González, **G.H.**, Seifert: in progress

Open String Axions & Open-Closed Mixing

- closed string axions = partners to Kähler & c.s. moduli
- **open string axions** can carry $U(1)_{PQ}$ charge: **QCD axion**

G.H., Staessens '13

- **limited choice** of $\boxed{U(1)_{PQ}}$
  $(\perp U(1)_Y = \frac{U(1)_a/3 + U(1)_c + U(1)_d}{2})$
  - $U(1)_b$ with $(\mathbf{1}_{\mathbf{Anti}_b})_2$ or $(\mathbf{1}_{\overline{\mathbf{Anti}_b}})_{-2}$
  - **or** $U(1)_{c-d}$ with $(\mathbf{1}^{cd})_{\pm 2}$
    $\leftrightarrow$ scalars $\tilde{\nu}_R$



- fits into **SUSY DFSZ axion model** with *twice* axion charge
  $\boxed{V = V_F + V_D + V_{\text{soft}}}$
- in the **explicit example** on $\mathbb{Z}_2 \times \mathbb{Z}_6$:

Ecker, G.H., Staessens '15

  - $U(1)_{PQ} \times \mathbb{Z}_3$ charges $\rightsquigarrow \mathcal{W}_{DFSZ} = \mu \Sigma H_u \cdot \tilde{H}_d + \tilde{\mu} \tilde{\Sigma} \tilde{H}_u \cdot H_d$
  - Σ also generates other mass terms: $\mathcal{W} \supset \kappa \overline{d}_R \Sigma d_R$

Open & Closed Axion Mixing in Field Theory

- open string axion a from $\sigma = \frac{v+s(x)}{\sqrt{2}} e^{i\frac{a(x)}{v}}$
- **open** axion a mixes with **closed** axion ξ ($\leftarrow U(1)_{\text{massive}}$)

$$\zeta_{\text{massive}} = \frac{M_{\text{string}}\,\xi + qv\,a}{\sqrt{M_{\text{string}}^2 + q^2v^2}}, \qquad \alpha_{\text{massless}} = \frac{M_{\text{string}}\,a - qv\,\xi}{\sqrt{M_{\text{string}}^2 + q^2v^2}}$$

- axion **decay constants** from dim. reduction:

$$f_\zeta = \frac{\sqrt{M_{\text{string}}^2 + (qv)^2}}{2}, \qquad f_\alpha = \frac{M_{\text{string}}\,qv\sqrt{M_{\text{string}}^2 + (qv)^2}}{(M_{\text{string}}^2 - (qv)^2)}$$

- For $\boxed{M_{\text{string}} \gg v}$ (field theory regime): $\zeta \simeq \xi_{\text{closed}}$, $\alpha \simeq a_{\text{open}}$

$\rightsquigarrow$ approximation of purely **open string as QCD axion** good

Conclusions & Outlook

- string theory as unification of gravity & QFT very **predictive**
  - Type II: endpoints of open strings constrain new physics
- need to re-combine **particle physics & cosmological** issues

  natural starting point: open/closed axions
- **partial** moduli **stabilisation** by D-branes
- **complete** moduli **stabilisation** might be very **unattractive**
  - fits to particle physics data
  - slow roll inflation
- **plethora** of stringy SM & GUT **spectra**, but missing: exact **field theory** results
  - tree level Yukawas on T^6/Γ
  - moduli dependence after deformation ...on generic CY_3
  - instanton corrections computationally intensive
  - **reliable** results on moduli potentials ...