

# Axions, Monodromy, and the 'Geometric Weak Gravity Conjecture'

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based on work with F. Rompineve/A. Westphal  
and on work in progress with J. Jaeckel/F. Rompineve/L. Witkowski  
and with P. Mangat/L. Witkowski/S. Theisen

## Outline

- The WGC beyond particles
- Dualities vs. the **geometric** WGC
- Constraining monodromy by the WGC for **domain walls**  

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- **Gravitational instantons** and the axion potential  

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- Reheating and **gravity waves** from axion monodromy

The WGC is interesting as ...

Arkani-Hamed/Motl/Nicolis/Vafa '06

## 1) A possible fundamental feature of quantum gravity

- It **quantifies** the non-existence of global symmetries

(If  $g \rightarrow 0$  is impossible, we need to know  $g_{min}$ .

The WGC states  $g_{min} = m$ .)

## 2) Since it may constrain large-field inflation / relaxation...

Cheung/Remmen; de la Fuente/Saraswat/Sundrum ... '14

Rudelius; Ibanez/Montero/Uranga/Valenzuela; Brown/Cottrell/Shiu/Soler;

Bachlechner/Long/McAllister; AH/Mangat/Rompineve/Witkowski;

Junghans; Heidenreich/Reece/Rudelius; Kooner/Parameswaran/Zavala;

Harlow; AH/Rompineve/Westphal; ... '15; Conlon/Krippendorf ... '16

Ibanez/Montero/Uranga/Valenzuela '15

## The (generalized) weak gravity conjecture

- The basic underlying lagrangian is  
(for  $p$ -dim. objects in  $d$  dims.; with  $\overline{M}_p \equiv 1$ )

$$S \sim \frac{1}{g^2} \int (F_{p+1})^2 + T \int_{p\text{-dim.}} dV + \int_{p\text{-dim.}} A_p$$

with

$$F_{p+1} = dA_p.$$

- To avoid stable extremal black branes, one requires charged objects with **sub-extremal** mass (tension):

$$q/T \geq \gamma_{p,d}^{1/2}, \quad \text{where} \quad \gamma_{p,d} = \frac{p(d-p-2)}{d-2}.$$

- As one clearly sees, this fails for **instantons** and objects with **codimension 1 & 2** (domain walls and cosmic 'strings').

## Note:

- This failure outside the range  $0 < p < d-2$  is not unexpected:
- Indeed, the argument that 'the WGC protects us from too many stable objects' fails also intuitively outside this range.

(E.g., strings and domain walls induce no long-range gravitational force.)

see e.g. Susskind '95

## However:

- The arguments that 'the WGC protects us from the global-symmetry limit' and 'string theory always obeys the WGC' support the conjecture even outside the above range.

- Arguments supporting/quantifying the WGC outside the 'canonical range' of  $0 < p < d-2$  include

- string dualities

Brown/Cottrell/Shiu/Soler '15

- consistency of generic KK-reductions
- consideration of dilatonic black branes.

Heidenreich/Reece/Rudelius '15  
(‘lattice WGC’)

- Here, we will try develop the duality argument....

- The key is **not** in the dualities, but rather in the fact that different  $p$ -branes on the **same CY** give different 4d objects.
- Hence, there ought to be a

### Geometric WGC

- Consider a IIA-CY  $X$  with D2-branes wrapped on 2-cycles.
- Let  $w_i$  be a basis of  $H^2(X, \mathbb{Z})$ .

The metric on  $X$  induces a metric for 2-forms,

$$K_{ij} \equiv \int_X w_i \wedge \star w_j,$$

and on the (dual) space of 2-cycles,  $K^{ij}$ .

- We make the standard ansatz

$$C_3 = A_1^i(x) \wedge w_i(y).$$

- Focus on 4d particles coming from D2s on a **particular** cycle  $\Sigma$
- The relevant 4d action reads ( $l_s = 1$ )

$$S_4 \sim (V_X/g_s^2) \int \sqrt{g} R + \int K_{ij} F_2^i \wedge \star F_2^j + q_i^\Sigma \int_{\text{world-line}} A_1^i$$

with the charges

$$q_i^\Sigma = \int_\Sigma w_i.$$

- Now, rewrite the WGC for particles in terms of compactification data:

$$\frac{e\bar{M}_P}{M_\Sigma} \geq \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \frac{|q^\Sigma| V_X^{1/2}}{V_\Sigma} \geq \frac{1}{2}.$$

Thus a particular, purely geometric (**rescaling- and  $g_s$ -independent**) quantity characterizing  $X$  is constrained.

- Crucially, the **same** function appears in WGC constraints on **other** objects obtained from **other** branes wrapped on 2-cycles.
- For example, D4s give domain walls with

$$\frac{e_{DW} \bar{M}_P}{T_{DW}} = \frac{(2V_X)^{1/2} |q^\Sigma|}{V_\Sigma}.$$

- Thus, using the ‘particle-WGC’, we constrain  $V_X^{1/2} |q^\Sigma| / V_\Sigma$ , obtaining a precise ‘domain-wall-WGC’:

$$\frac{e_{DW} \bar{M}_P}{T_{DW}} \geq \frac{1}{\sqrt{2}}.$$

- This goes through for **any** dimension of the cycle  $\Sigma$  and **any** dimension of the brane. Hence, **any** object in 4d is constrained by the imposition of the WGC for particles.



- Thus, allowing also for multiple gauge fields,

Cheung/Remmen '14; Rudelius '14/'15,  
Brown/Cottrell/Shiu/Soler, Bachlechner/Long/McAllister '15

we find **in full generality**:

Geometric conjecture:

The convex hull spanned by the vectors  $(V_X^{1/2}/V_\Sigma) q^\Sigma$   
(with  $\Sigma \in H^p(X, \mathbb{Z})$ ) contains the ball of radius  $1/2$ .

- Note: At the structural level, this can be understood from the calibration condition on branes. Details remain to be worked out....

Thanks to F. Marchesano for discussions on this point.

- Note: We did not use SUSY, the CY-condition, or the existence of a SUSY-brane on  $\Sigma$ . So this may be much stronger than the 'not too surprising' BPS-like result.

see also work in progress by Heidenreich/Reece/Rudelius

## Constraining axion monodromy with the WGC

### Disclaimer:

Only brief summary; for deeper analysis and relation to earlier work...

Kaloper/Lawrence/Sorbo '08..'11 (see also Dvali '05)

Brown/Cottrell/Shiu/Soler; Ibanez/Montero/Uranga/Valenzuela '15

- Let's assume, based on the above, that all 4d objects, in particular DWs, are constrained.
- Note: the 'light' stringy objects fulfilling the WGC above are nevertheless always heavier than the KK-scale  $M_{KK} = \Lambda$ .
- Thus, one might conjecture that the magnetic WGC

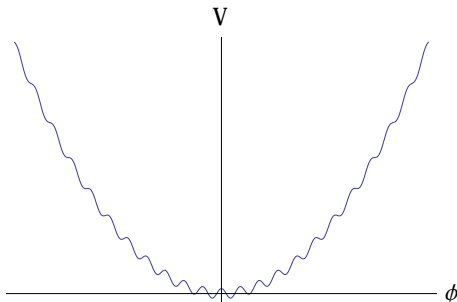
$$\Lambda^3 \lesssim e_2 \overline{M}_P$$

always holds.

- Start from the 'standard' monodromy potential (with 'instantonic wiggles')

AH/Rompineve/Westphal '15

$$\mathcal{L} = (\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \alpha \cos(\varphi/f).$$



The low-energy effective theory of this model has no scalar but just a set of discrete vacua (as in the Bousso-Polchinski landscape).

(Effective) domain walls are automatically present, but are too light to give any useful WGC constraint.

(In fact, one may argue that they make the electric WGC useless.)

- Nevertheless, the effective action

$$S \sim \int \frac{1}{2(e_2)^2} F_4^2 + \int_{DW} A_3$$

is there and, using the quantization  $F_4 = n e_2^2$ , allows for matching the discrete effective potential

$$V(F_4)_{eff} = \frac{1}{2}(e_2)^2 n^2$$

to the previous effective potential

$$V(\varphi)_{eff} = \frac{1}{2} m^2 (2\pi n f)^2.$$

- This implies  $e_2 = 2\pi m f$  and hence

$$\Lambda^3 \lesssim e_2 \overline{M}_P = 2\pi m f \overline{M}_P.$$

- In the context of inflation, one has

$$H \sim m \varphi_{\max} \lesssim \Lambda$$

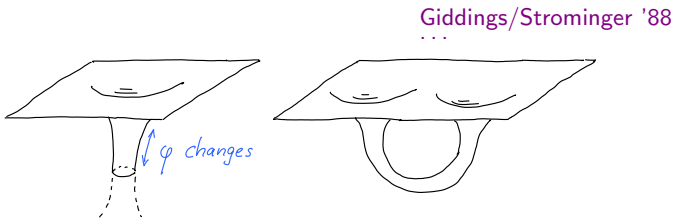
and hence

$$\Lambda^3 \sim m f \overline{M}_P \quad \Rightarrow \quad \frac{\varphi_{\max}}{\overline{M}_P} \lesssim \left( \frac{\overline{M}_P}{m} \right)^{2/3} \left( \frac{2\pi f}{\overline{M}_P} \right)^{1/3} .$$

- There is still lots of parameter room for large-field inflation....

## Gravitational Instantons and Moduli Stabilization

- In Euclidean Einstein gravity, supplemented with an axionic scalar  $\varphi$ , instantonic solutions exist:



- The 'throat' is supported by the gradient energy of  $\varphi$  or, equivalently, by flux of the dual 3-form  $H_3$ .
- The relevance for inflation arises through the induced instanton-potential for the originally **shift-symmetric** field  $\varphi$ .

Montero/Uranga/Valenzuela '15  
Heidenreich/Reece/Rudelius '15

- The instanton action is

$$S \sim n/f \quad (\text{with } n \text{ the instanton or flux number}).$$

- Their maximal curvature scale is  $\sqrt{f/n}$ , which should not exceed the UV cutoff:

$$f/n < \Lambda^2.$$

- This fixes the lowest  $n$  that we can trust and hence the minimal size of the instanton correction to the potential  $V(\varphi)$ :

$$\delta V \sim e^{-S} \sim e^{-n/f} \sim e^{-1/\Lambda^2}$$

- For gravitational instantons **not** to prevent inflation, the **relative** correction must remain small:

$$\frac{\delta V}{V} \sim \frac{e^{-1/\Lambda^2}}{H^2} \ll 1$$

- For a Planck-scale cutoff,  $\Lambda \sim 1$ , this is never possible
- However, the UV cutoff can in principle be as low as  $H$
- Then, if also  $H \ll 1$ , everything might be fine....

$$\frac{\delta V}{V} \sim \frac{e^{-1/H^2}}{H^2}$$

AH, Mangat, Rompineve, Witkowski '15



## Results to appear soon:

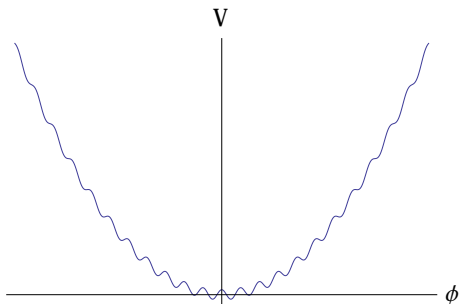
work with Mangat/Theisen/Witkowski

- Coleman's calculation of the potential remains valid even though one always encounters instanton / anti-instanton pairs.
- The size of the effect does not get suppressed by  $\exp(-1/\Lambda^2)$ , with  $\Lambda$  the moduli scale.  
(Light moduli do not disturb the solution significantly.)
- Hence, we expect  $\Lambda \sim m_{KK}$ .
- Let us see what the **strongest, model-independent** bound is:  
(Taking  $\Lambda = 1/R_{throat} = 1/R_{self-dual}$ .)
- Maximal effect:  $\exp(-S) = \exp(-3\pi^2) \sim 10^{-13}$ .

→ parallel talks by L. Witkowski and P. Mangat

## Gravity Waves from Monodromy

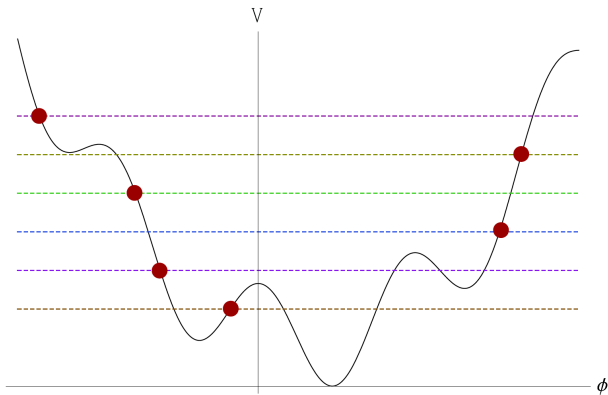
(work in progress with Jaeckel/Rompineve/Witkowski)



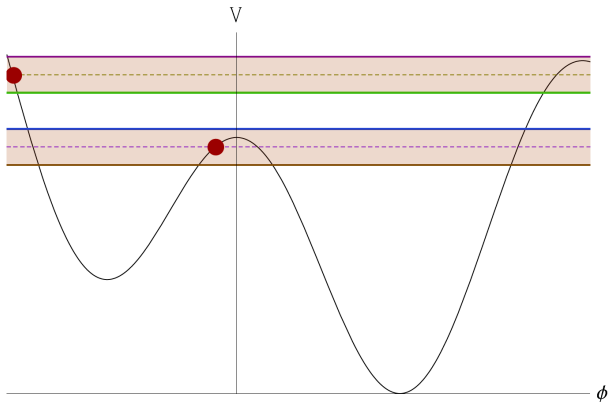
How does Repeating in this potential work?

for (somewhat) related considerations see papers by  
T. Higaki and F. Takahashi (with different collaborators);  
Kaloper/Padilla '16; Jaeckel/Metha/Witkowski '16

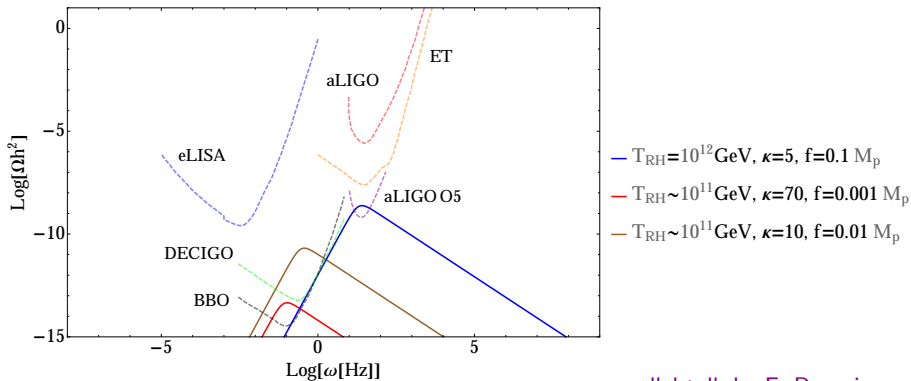
- The field oscillates and eventually 'gets stuck' in one of the local minima
- It then continues to oscillate in that minimum (where it later decays to light particles, i.e. reheats)



- At each 'turning point', an uncertainty due to field fluctuations exists
- Hence, with a certain probability, two different minima are populated inside one Hubble patch



- Eventually, bubbles of the lowest populated minimum expand and collide
- Gravity waves are produced in analogy to the case of a thermal first-order phase transition



→ parallel talk by F. Rompineve

## Summary (1)

- Let's assume that string compactifications with form-fields / wrapped objects always obey the **particle WGC**.
- Then a **geometric WGC** follows.
- From this, one obtains a **generalized WGC** including axions, cosmic strings and DWs etc.
- The KK scale is always so low that also the **generalized magnetic WGC** is holds.  
Let's accept this latter form also more generally.
- The magnetic WGC for DWs provides for a very direct way of **constraining axion-monodromy-type scalar potentials**.

## Summary (2)

- Independently of the WGC, Giddings-Strominger wormholes constrain large-field inflation
- This effect persists above the moduli stabilization scale; Calculational control is only lost at the KK scale
- However, due to a surprisingly large ' $3\pi^2$ ' prefactor, bounds are weak even for the highest possible KK scale

## Summary (3)

- Reheating after axion monodromy or 'winding' inflation can lead to a '**dynamical phase decomposition**'
- This can induce a rather significant gravity wave signal