Starobinsky-Type Inflation With $SU(1,1)/U(1) \times SU(2)/U(1)$ Kähler Manifolds

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Based on:

- C.P. & N. Toumbas, *J. Cosmol. Astropart. Phys.* **05**, *no. 05, 019 (2016)* [arXiv:1512.05657].

Outline

Starobinsky-Type Inflation (STI)
Non-Minimal Inflation (nMI)
Starobinsky Versus Induced-Gravity Inflation
The SUGRA Framework
The General Set-Up
The Idea of Induced Gravity in SUGRA
Kähler Potentials
STI and Kähler Potentials
Kähler Potentials Vs Kähler Manifolds
Inflationary Observables - Results
Conclusions

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Coupling non-Minimally the Inflaton to Gravity

• Our Starting Point is The Action in the **Jordan Frame (JF)** Of A Scalar Field ϕ with Potential $V(\phi)$ non-Minimally Coupled to the Ricci Scalar Curvature, $\mathcal{R}$, Through A Frame Function $f_{\mathcal{R}}(\phi)$. This is:

$$\mathcal{S} = \int d^4x \sqrt{-\mathfrak{g}} \left(-\frac{1}{2} f_{\mathcal{R}}(\phi)\mathcal{R} + \frac{f_{\rm K}(\phi)}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \right), \quad \text{Where}$$

$\mathfrak{g}$ is the Determinant Of The Background Metric and $f_{\mathcal{R}}(\langle\phi\rangle) \simeq 1$ (in Reduced Planck Units With $m_{\rm P} = 1$) to Guarantee the Ordinary **Einstein Gravity** At Low Energy. We Allow for a Kinetic Mixing Through the Function $f_{\rm K}(\phi)$.

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• If we Perform a **Conformal Transformation**[1] According Which We Define the EF Metric:

$$\widehat{g}_{\mu\nu} = f_{\mathcal{R}}\, g_{\mu\nu} \quad \Rightarrow \quad \begin{cases} \sqrt{-\widehat{\mathfrak{g}}} = f_{\mathcal{R}}^2 \sqrt{-\mathfrak{g}} \quad \text{and} \quad \widehat{g}^{\mu\nu} = g^{\mu\nu}/f_{\mathcal{R}}, \\ \widehat{\mathcal{R}} = \left(\mathcal{R} + 3\Box \ln f_{\mathcal{R}} + 3 g^{\mu\nu} \partial_\mu f_{\mathcal{R}} \partial_\nu f_{\mathcal{R}} / 2 f_{\mathcal{R}}^2 \right) / f_{\mathcal{R}} \end{cases}$$

We End up with the Action $\mathcal{S}$ in the **Einstein Frame (EF)**

$$\mathcal{S} = \int d^4x \sqrt{-\widehat{\mathfrak{g}}} \left(-\frac{1}{2} \widehat{\mathcal{R}} + \frac{1}{2} \widehat{g}^{\mu\nu} \partial_\mu \widehat{\phi} \partial_\nu \widehat{\phi} - \widehat{V}\left(\widehat{\phi}\right) \right)$$

Where we Introduce the **EF Canonically Normalized Field**, $\widehat{\phi}$, and **Potential**, $\widehat{V}$, Defined As Follows:

$$\left(\frac{d\widehat{\phi}}{d\phi} \right)^2 = J^2 = \frac{f_{\rm K}}{f_{\mathcal{R}}} + \frac{3}{2} \left(\frac{f_{\mathcal{R},\phi}}{f_{\mathcal{R}}} \right)^2 \quad \text{and} \quad \widehat{V}(\widehat{\phi}) = \frac{V\left(\widehat{\phi}(\phi)\right)}{f_{\mathcal{R}}\left(\widehat{\phi}(\phi)\right)^2}.$$

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- We Observe that $f_{\mathcal{R}}$ Affects **Both** J and $\widehat{V}_{\rm I}$ whereas $f_{\rm K}$ only J;
- Obviously a clever Choice of V and $f_{\mathcal{R}}$ Can Lead to A **Plateau** Convenient for Drive Inflation;
- The Analysis of Inflation in **the EF** Using The Standard Slow-Roll Approximation is Equivalent With **The Analysis in JF**.

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Inflationary Observational and Theoretical Requirements

In the Era of Precision Cosmology The Inflationary Particle Models Can Be Tightly Restricted.

- The **Number of e-foldings**, $\widehat{N}_\star$, that the Scale $k_\star = 0.05/\mathrm{Mpc}$ Suffers During inflation has to be Sufficient to Resolve the Horizon and Flatness Problems of Standard Big Bang:

$$\widehat{N}_\star = \int_{\widehat{\phi}_\mathrm{f}}^{\widehat{\phi}_\star} d\widehat{\phi}\, \frac{\widehat{V}}{\widehat{V}_{,\widehat{\phi}}} = \int_{\phi_\mathrm{f}}^{\phi_\star} d\phi\, J^2\, \frac{\widehat{V}}{\widehat{V}_{,\phi}} \simeq 50 - 60$$

Where $\widehat{V}$ is the EF scalar potential of the Inflaton ϕ;
$\phi_\star$ $[\widehat{\phi}_\star]$ is The Value of ϕ $[\widehat{\phi}]$ **When $k_\star$ Crosses Outside** The Inflationary Horizon;
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$$\max\{\widehat{\epsilon}(\phi_\mathrm{f}), |\widehat{\eta}(\phi_\mathrm{f})|\} = 1, \quad \text{With } \widehat{\epsilon} = \frac{1}{2}\left(\frac{\widehat{V}_{,\widehat{\phi}}}{\widehat{V}}\right)^2 = \frac{1}{2J^2}\left(\frac{\widehat{V}_{,\phi}}{\widehat{V}}\right)^2 \text{ and } \widehat{\eta} = \frac{\widehat{V}_{,\widehat{\phi}\widehat{\phi}}}{\widehat{V}} = \frac{1}{J^2}\left(\frac{\widehat{V}_{,\phi\phi}}{\widehat{V}} - \frac{\widehat{V}_{,\phi}}{\widehat{V}}\frac{J_{,\phi}}{J}\right).$$

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- The **Amplitude A_s of the Power Spectrum** of the Curvature Perturbations is To Be Normalized With ***Planck*** Data:

$$A_\mathrm{s}^{1/2} = \frac{1}{2\sqrt{3}\,\pi}\,\frac{\widehat{V}(\widehat{\phi}_\star)^{3/2}}{|\widehat{V}_{,\widehat{\phi}}(\widehat{\phi}_\star)|} = \frac{|J(\phi_\star)|}{2\sqrt{3}\,\pi}\,\frac{\widehat{V}(\phi_\star)^{3/2}}{|\widehat{V}_{,\phi}(\phi_\star)|} = 4.627\cdot 10^{-5}\,.$$

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- We Have To Check The Hierarchy Between **The Ultraviolet Cut-off**[2], Λ_UV, of the Effective Theory And The Inflationary Scale. In Particular, The Validity Of The Effective Theory Implies:

$$\text{(a)}\ \ \widehat{V}(\phi_*)^{1/4} \leq \Lambda_\mathrm{UV} \ \text{ for } \ \text{(b)}\ \ \phi \leq 1$$

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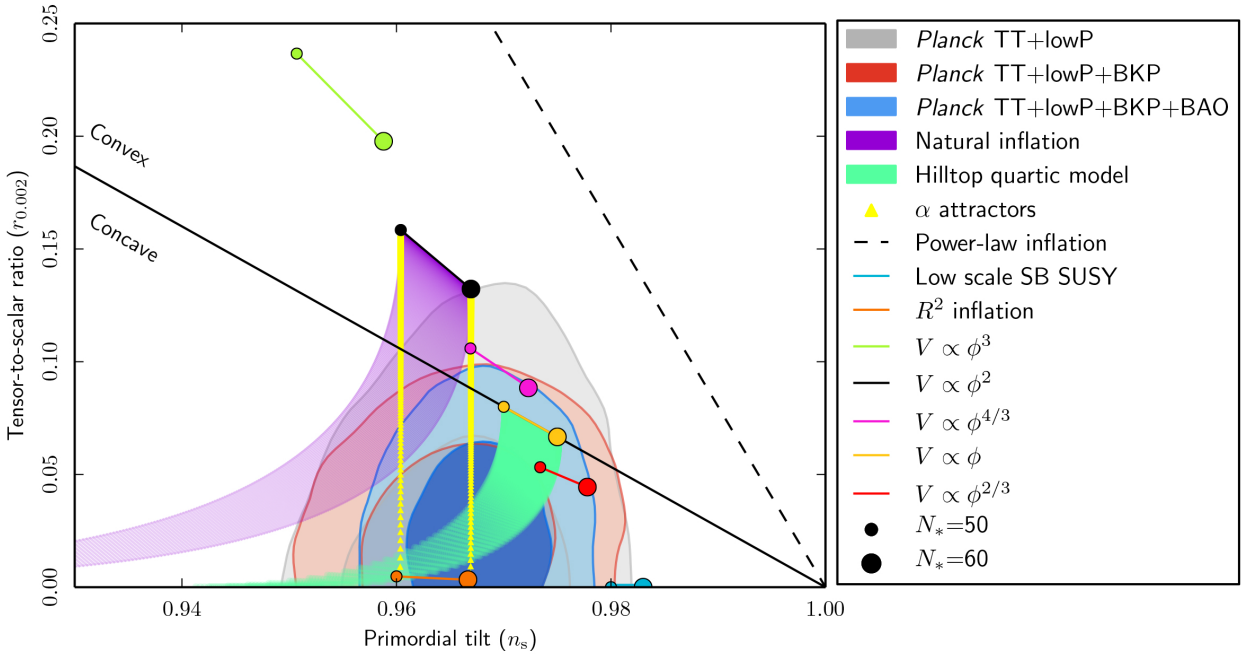
Constraints for the ΛCDM + r Model From BICEP2/*Keck Array* and *Planck* 2015



• Inflationary Models Which Succeed to Fit the Observational Data on A_s and $N_\star$ Can Be Further Restricted If We Calculate The (Scalar) **Spectral Index** And **Tensor-to-Scalar Ratio** Found Respectively As:

$$n_s = 1 - 6\epsilon(\phi_\star) + 2\eta(\phi_\star) \quad \text{and} \quad r = 16\epsilon(\phi_\star)$$

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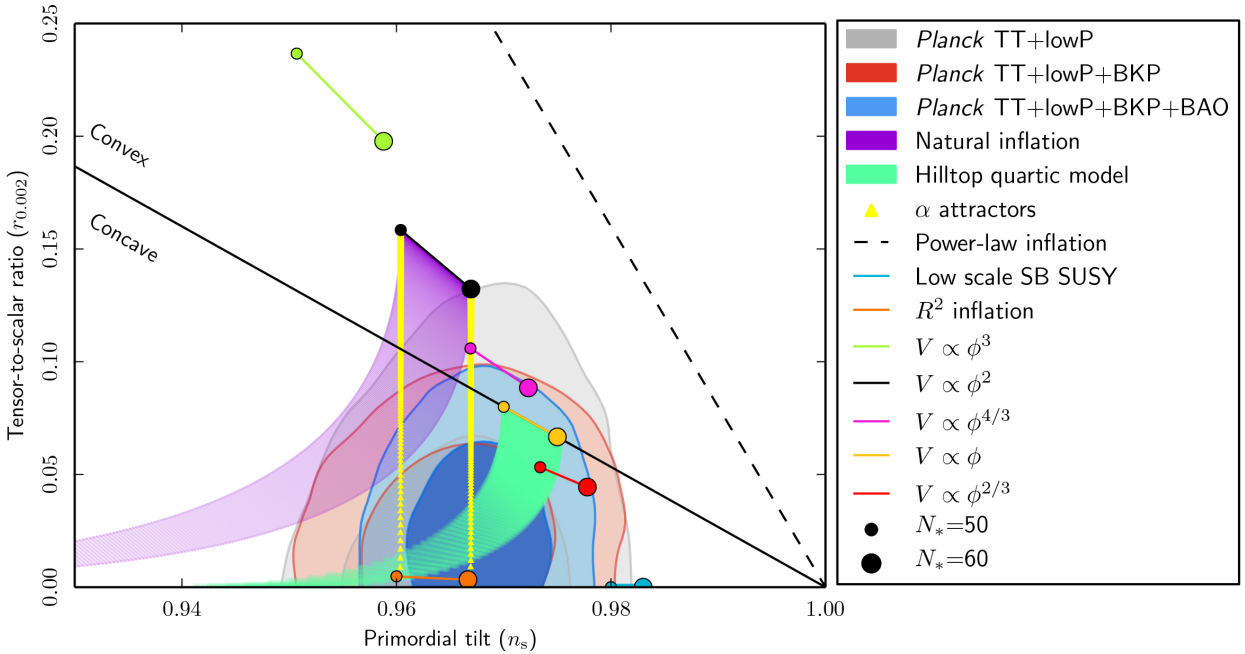

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- The Combined BICEP2/*Keck Array* and *Planck* Results Yield

$$n_s = 0.968 \pm 0.009 \quad \text{and} \quad r = 0.028^{+0.026}_{-0.025} \quad \Rightarrow \quad 0.003 \lesssim r \lesssim 0.054 \quad \text{at 68\%c.l. or} \quad r \lesssim 0.07 \quad \text{at 95\%c.l.}$$

I.e., Inflationary Models With Negligible r's Are Not Excluded, Although Those With r's of order 0.01 Seem To Be Favored.

- R^2 Inflation (Or **Starobinsky** Inflation) **Predicts $n_s \simeq 0.964$ and $r = 0.003$** for $\widehat{N}_\star \simeq 52$.
- As A Consequence, the Starobinsky Inflation Remains One of the **Most Predictive and Successful** Models.

From Non-Minimal to $\mathcal{R}^2$ Inflation

- The $\mathcal{R}^2$ Inflation can be Introduced as a Type of nMI Employing an **Auxiliary** (Non-Propagating) Field ϕ With

$$f_K = 0, \quad f_{\mathcal{R}} = 1 + 4c_{\mathcal{R}}\phi \quad \text{And} \quad V = \phi^2$$

Using The Equation Of Motion $\phi = c_{\mathcal{R}}\mathcal{R}$ We Obtain The Action Of The **Original** Model:

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$$\widehat{V} = \frac{\phi^2}{f_{\mathcal{R}}^2} \simeq \frac{1}{16{c_{\mathcal{R}}}^2}, \ \widehat{\epsilon} \simeq \frac{1}{12{c_{\mathcal{R}}}^2\phi^2} \quad \text{and} \quad \widehat{\eta} \simeq \frac{1-4c_{\mathcal{R}}\phi}{12{c_{\mathcal{R}}}^2\phi^2}.$$

  - Therefore, $\max\{\widehat{\epsilon}(\phi_{\rm f}), |\widehat{\eta}(\phi_{\rm f})|\} = 1 \ \Rightarrow \ \phi_{\rm f} = \dfrac{1}{2\sqrt{3}c_{\mathcal{R}}}$.

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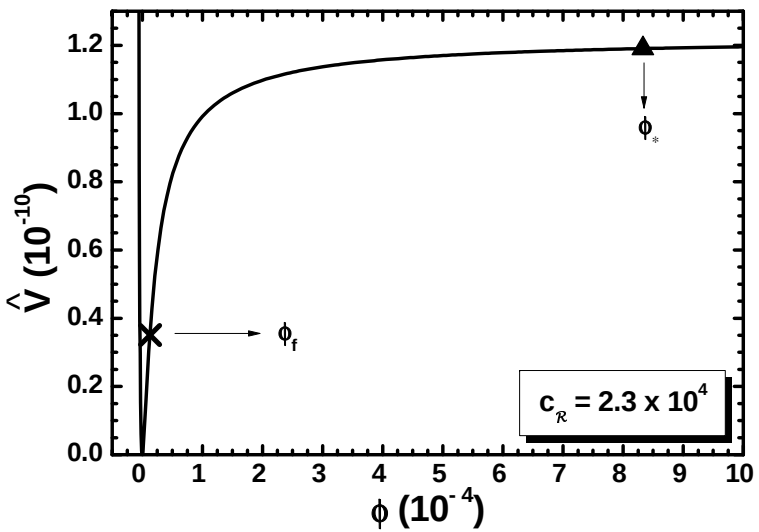

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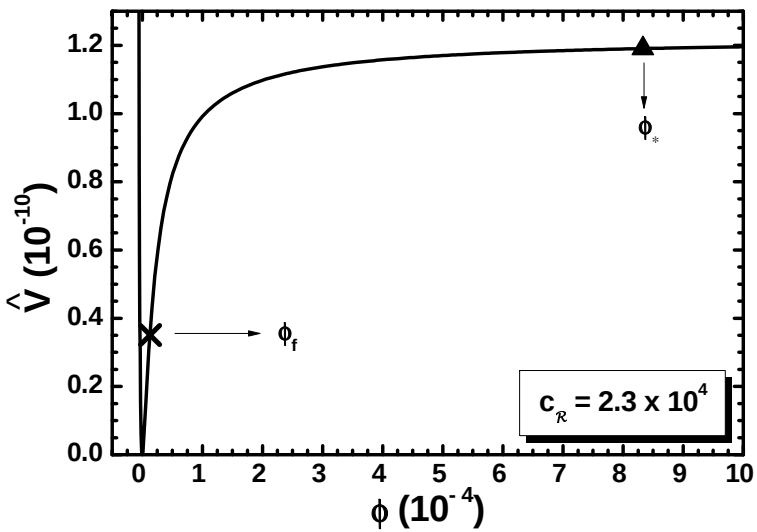

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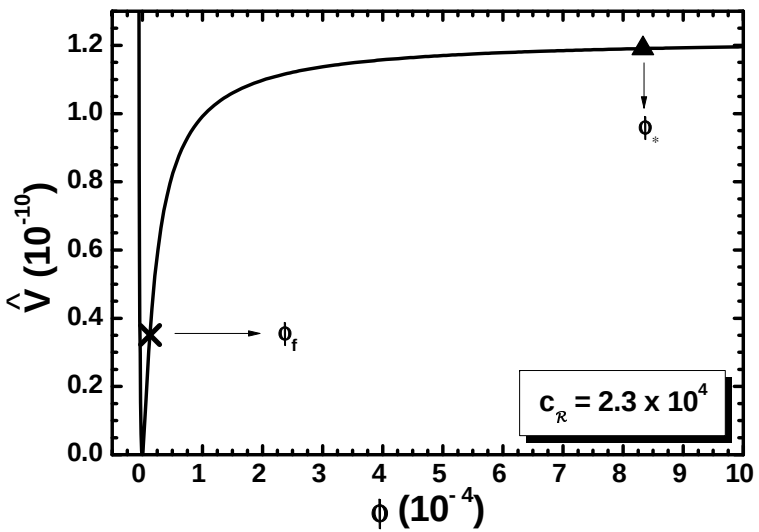

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- There is **No Problem with Perturbative Unitarity**, Since We Obtain $\overline{\Lambda_{\rm UV}=1}$ If we Perform Expansions Around $\langle\phi\rangle=0$:

$$J^2\dot{\phi}^2=\left(1-2\sqrt{\frac{2}{3}}\widehat{\phi}+2\widehat{\phi}^2-\cdots\right)\dot{\widehat{\phi}}^2\quad\text{and}\quad\widehat{V}=\frac{\widehat{\phi}^2}{24{c_{\mathcal{R}}}^2}\left(1-2\sqrt{\frac{2}{3}}\widehat{\phi}+2\widehat{\phi}^2-\cdots\right)\ \text{With}\ \widehat{\phi}=2\sqrt{3}c_{\mathcal{R}}\phi$$

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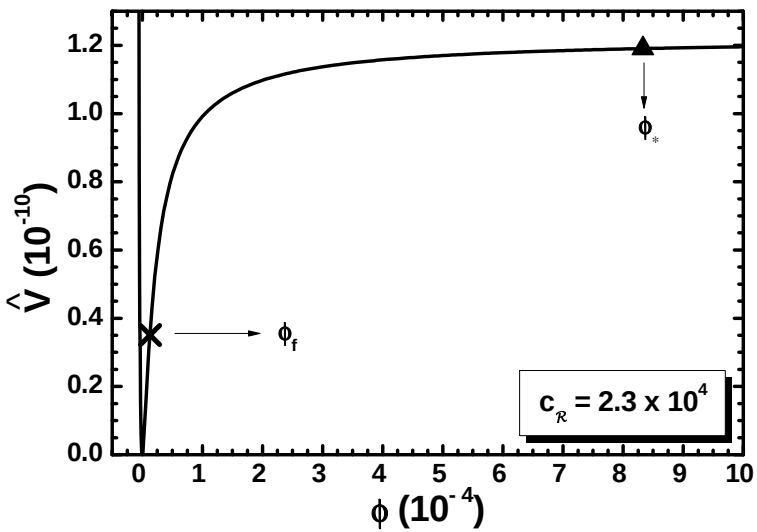

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- $n_{\rm s}\simeq 1-2/\widehat{N}_\star\simeq 0.965$, $\alpha_{\rm s}\simeq-2/\widehat{N}_\star^2\simeq-6.4\cdot10^{-4}$ And $r\simeq 12/\widehat{N}_\star^2\simeq 4\cdot10^{-3}$ **(In Agreement With Observations).**

- There is **No Problem with Perturbative Unitarity**, Since We Obtain $\overline{\Lambda_{\rm UV}=1}$ If we Perform Expansions Around $\langle\phi\rangle=0$:

$$J^2\dot{\phi}^2=\left(1-2\sqrt{\frac{2}{3}}\widehat{\phi}+2\widehat{\phi}^2-\cdots\right)\dot{\widehat{\phi}}^2\quad\text{and}\quad\widehat{V}=\frac{\widehat{\phi}^2}{24{c_{\mathcal{R}}}^2}\left(1-2\sqrt{\frac{2}{3}}\widehat{\phi}+2\widehat{\phi}^2-\cdots\right)\ \text{With}\ \widehat{\phi}=2\sqrt{3}c_{\mathcal{R}}\phi$$

- The **Mass of the Inflaton** at the Vacuum is: $\widehat{m}_{\delta\phi}=\left\langle\widehat{V}_{,\widehat{\phi}\widehat{\phi}}\right\rangle^{1/2}=\left\langle\widehat{V}_{,\phi\phi}/J^2\right\rangle^{1/2}=1/2\sqrt{3}c_{\mathcal{R}}\simeq 1.25\cdot10^{-5}$ (i.e. $3\cdot10^{13}$ GeV).

Induced-Gravity Inflation (IGI)

- It Would Be Certainly Beneficial to Obtain STI Avoiding Drastic Deviations from Einstein Gravity, at Least at Present.

For this reason we Introduce the Idea of **Induced Gravity**.

- IGI Can Be Realized Employing an **Real-Propagating** Field ϕ If We

Adopt The Following Ingredients:

$$f_K = 1, \quad f_{\mathcal{R}} = c_{\mathcal{R}}\phi^2 \quad \text{and} \quad V = \lambda\left(\phi^2 - M^2\right)^2/4\,.$$

- **Recovering Einstein Gravity** at the Vacuum Implies

$$f_{\mathcal{R}}(\langle\phi\rangle) = 1 \quad \Rightarrow \quad M = 1/\sqrt{c_{\mathcal{R}}}.$$

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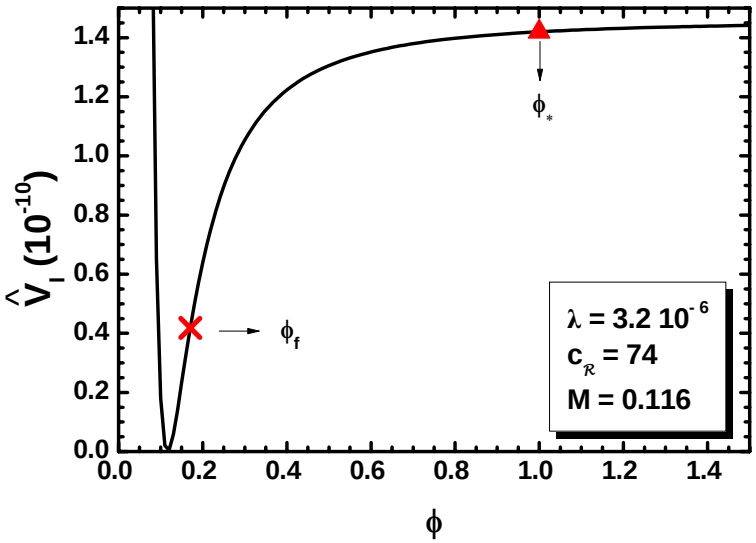

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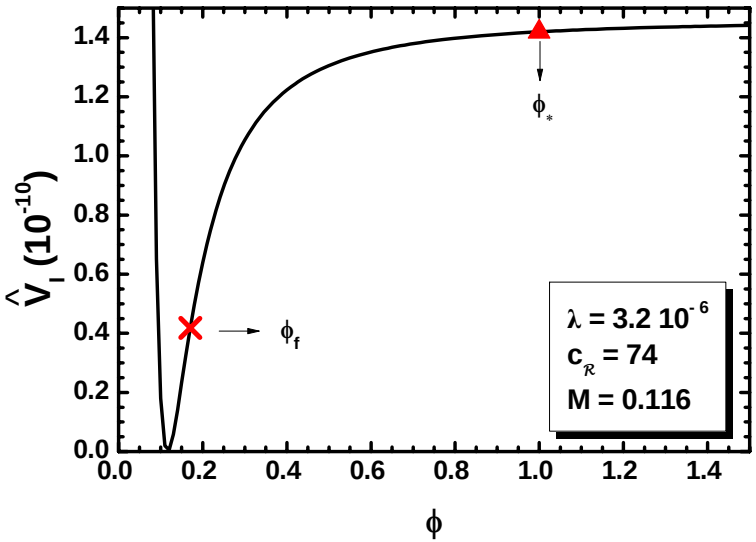

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SUGRA Scalar Potential in EF & JF (For a Gauge Singlet Inflaton)

- The General **EF** Action For The Scalar Fields Φ^α Plus Gravity In Four Dimensional $\mathcal{N}=1$ SUGRA is:

$$\mathcal{S}=\int d^4x\sqrt{-\widehat{\mathfrak{g}}}\left(-\frac{1}{2}\widehat{\mathcal{R}}+K_{\alpha\bar{\beta}}\widehat{g}^{\mu\nu}\partial_\mu\Phi^\alpha\partial_\nu\Phi^{*\bar{\beta}}-\widehat{V}\right)\ \text{Where}\ \widehat{V}=\widehat{V}_{\rm F}=e^K\left(K^{\alpha\bar{\beta}}(D_\alpha W)(D^*_{\bar{\beta}}W^*)-3|W|^2\right),$$

K is The Kähler Potential With $K_{\alpha\bar{\beta}}=\dfrac{\partial^2 K}{\partial\Phi^\alpha\partial\Phi^{*\bar{\beta}}}>0$ and $K^{\alpha\bar{\beta}}K_{\alpha\bar{\gamma}}=\delta^{\bar{\beta}}_{\bar{\gamma}};\quad D_\alpha W=W_{,\Phi^\alpha}+K_{,\Phi^\alpha}W.$

Therefore, Implementing STI Within SUGRA Requires The Appropriate Selection of W and K

[3] *M.B. Einhorn and D.R.T. Jones (2010); S. Ferrara et al. (2010, 2011); H.M. Lee (2010); C.P. and N. Toumbas (2011).*

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- If We Set [3] $K=-N\ln\left(-\Omega/N\right)$ and Perform a Conformal Transformation, $\mathcal{S}$ **In JF** Reads

$$\mathcal{S}=\int d^4x\sqrt{-\mathrm{g}}\left(\frac{\Omega}{2N}\mathcal{R}+\left(\Omega_{\alpha\bar{\beta}}+\frac{3-N}{N}\frac{\Omega_\alpha\Omega_{\bar{\beta}}}{\Omega}\right)\partial_\mu\Phi^\alpha\partial^\mu\Phi^{*\bar{\beta}}-\frac{27}{N^3}\Omega\mathcal{A}_\mu\mathcal{A}^\mu-V\right),\quad\textbf{Ω: Frame Function}$$

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- We Observe that Ω Enters The Kinetic Terms of the Φ^α's too. $\mathcal{S}$ **Can Exhibit Non-Minimal Couplings** of Φ^α's to $\mathcal{R}$ If
  - $\mathcal{A}_\mu=0$ Where $\mathcal{A}_\mu=-iN\left(\partial_\mu\Phi^\alpha\Omega_\alpha-\partial_\mu\Phi^{*\bar\alpha}\Omega_{\bar\alpha}\right)/6\Omega$. This Happens When $\arg\Phi^\alpha=0$ or $\Phi^\alpha=0$ During Inflation;
  - We Can Decompose Ω to an **Holomorphic** $\Omega_{\mathrm{H}}=\Omega_{\mathrm{H}}(\Phi^\alpha)$ and a **Kinetic** (Real) $\Omega_{\mathrm{K}}=\Omega_{\mathrm{K}}\left(\Phi^\alpha\Phi^{*\bar\beta}\right)$ Part, With $\Omega_{\mathrm{H}}\gg\Omega_{\mathrm{K}}\simeq\delta_{\alpha\bar\beta}\Phi^\alpha\Phi^{*\bar\beta}$ Where We Restrict Ourselves to the Lowest Order **Quadratic Terms**. Therefore

$$\Omega=\Omega_{\mathrm{K}}-N\left(\Omega_{\mathrm{H}}(\Phi^\alpha)+\Omega^*_{\mathrm{H}}(\Phi^{*\bar\alpha})\right)\ \Rightarrow\ K=-N\ln\left(\Omega_{\mathrm{H}}(\Phi^\alpha)+\Omega^*_{\mathrm{H}}(\Phi^{*\bar\alpha})-\Omega_{\mathrm{K}}/N\right).$$

  - Although $N=3$ is Standard Since it Assures Canonical Terms for Φ_α's, $0<N\neq3$ is Totally Acceptable.

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Selecting Conveniently the Superpotential and Frame Function

- **Recovering Einstein Gravity** At Low Energies Dictates $\langle \Omega_{\rm H} \rangle + \langle \Omega_{\rm H}^{*} \rangle = 1 \Rightarrow \langle \Omega_{\rm H} \rangle = 1/2$ for $\langle \Omega_{\rm H} \rangle = \langle \Omega_{\rm H}^{*} \rangle$ and $\langle \Omega_{\rm K} \rangle \sim 0$.
- We Use 2 Superfields $\Phi^1 = T$ (**Inflaton**) and $\Phi^2 = S$ (**"Stabilizer" Field**) And Adopt the Following Superpotential:

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  - $K_{21} = -n_{21} \ln\left(f(T) + f^*(T^*) - \frac{|S|^2}{n_{21}}\right)$ Which Parameterizes the $SU(2,1)/SU(2) \times U(1)$ Kähler Manifold.

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  - $K_{111} = -n_{11} \ln\left(f(T) + f^*(T^*)\right) + |S|^2$ Which Parameterizes the $SU(1,1)/U(1) \times U(1)$ Kähler Manifold.

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$$\widehat{V}_{\rm I0} \simeq \lambda^2 K^{SS^*} / c_T^2 f_{\mathcal{R}}^N \quad \text{Where} \quad f_{\mathcal{R}} = -\Omega/N, \quad \text{and} \quad e^K = f_{\mathcal{R}}^{-N}.$$

- As Regards the S terms in K We Distinguish The Following **Cases:**
  - $K_{21} = -n_{21} \ln\left(f(T) + f^*(T^*) - \frac{|S|^2}{n_{21}}\right)$ Which Parameterizes the $SU(2,1)/SU(2) \times U(1)$ Kähler Manifold.
  - $K_{111} = -n_{11} \ln\left(f(T) + f^*(T^*)\right) + |S|^2$ Which Parameterizes the $SU(1,1)/U(1) \times U(1)$ Kähler Manifold.
  - $K_{112} = -n_{11} \ln\left(f(T) + f^*(T^*)\right) + n_2 \ln\left(1 + \frac{|S|^2}{n_2}\right)$ Which Parameterizes the $SU(1,1)/U(1) \times SU(2)/U(1)$ Kähler Manifold.

Selecting Conveniently the Superpotential and Frame Function

- **Recovering Einstein Gravity** At Low Energies Dictates $\langle \Omega_{\rm H} \rangle + \langle \Omega_{\rm H}^* \rangle = 1 \Rightarrow \langle \Omega_{\rm H} \rangle = 1/2$ for $\langle \Omega_{\rm H} \rangle = \langle \Omega_{\rm H}^* \rangle$ and $\langle \Omega_{\rm K} \rangle \sim 0$.
- We Use 2 Superfields $\Phi^1 = T$ (**Inflaton**) and $\Phi^2 = S$ (**"Stabilizer" Field**) And Adopt the Following Superpotential:

$$W = \frac{\lambda}{c_T} S\,(\Omega_{\rm H} - 1/2) \quad \text{with} \quad \Omega_{\rm H}(T) = c_T T^n + \sum_{k=2}^{\infty} \lambda_k T^{nk} \simeq c_T T^n = f(T) \quad \text{for} \quad T < 1 \quad \text{and} \quad \lambda_k \ll 1$$

- W Is Invariant Under A Global $\mathbb{Z}_n$ Symmetry and Respects a Continuous R Symmetry, i.e.,

$$W \to W \quad \text{for} \quad T \to e^{2ik\pi/n} T\,, \quad \text{and} \quad W \to e^{i\varphi} W \quad \text{for} \quad S \to e^{i\varphi} S \quad \text{and} \quad \Omega_{\rm H} \to \Omega_{\rm H}\,.$$

- The SUSY limit, $V_{\rm SUSY}$, of $\widehat{V}$, obtained for $m_{\rm P} \to \infty$ is $V_{\rm SUSY} = \lambda^2 \left|\Omega_{\rm H} - 1/2\right|^2 / c_T^2 + \lambda^2 |S\,\Omega_{{\rm H},\Phi}|^2 / c_T^2$.
- The SUSY Vacuum Lies At The Direction $\langle S \rangle = 0$ and $\langle \Omega_{\rm H} \rangle = 1/2 \;\Rightarrow\; \langle T \rangle = (2c_T)^{-1/n}$.

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In The Following We Show Details on the Realization of These Three Models.

STI & Curvature of the Kähler Manifolds

- Expanding Φ and S in Real and Imaginary Parts as Follows:

$$T = \frac{\phi e^{i\theta}}{\sqrt{2}} \quad \text{and} \quad S = \frac{s + i\bar{s}}{\sqrt{2}}, \quad \text{The Inflationary Direction is Defined as} \quad s = \bar{s} = \theta = 0.$$

- We Can Introduce The EF Canonically Normalized Fields, so as

$$K_{\alpha\bar{\beta}}\dot{z}^{\alpha}\dot{z}^{*\bar{\beta}} = \frac{1}{2}\left(\dot{\widehat{\phi}}^2 + \dot{\widehat{\theta}}^2\right) + \frac{1}{2}\left(\dot{\widehat{s}}^2 + \dot{\widehat{\bar{s}}}^2\right) \quad \text{Where} \quad \frac{d\widehat{\phi}}{d\phi} = J = \sqrt{K_{TT^*}}, \quad \widehat{\theta} = J\,\frac{\theta}{\phi}, \quad \text{and} \quad (\widehat{s}, \widehat{\bar{s}}) = \sqrt{K_{SS^*}}(s, \bar{s}).$$

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- Along the inflationary Trajectory, $\widehat{V}_{\rm I}$ is Written As

$$\widehat{V}_{\rm I} = \frac{\lambda^2(1-2f)^2}{c_T^2}\cdot\begin{cases}2^{-(1+n_{21})}/f^{n_{21}-1}\\ 2^{-(2+n_{11})}/f^{n_{11}}\end{cases} \quad \text{Since} \quad \left(K_{\alpha\bar{\beta}}\right) = \text{diag}\left(K_{TT^*}, K_{SS^*}\right) = \begin{cases}\text{diag}\left(\frac{n_{21}n^2}{2\phi^2}, \frac{2^{n/2-1}}{f}\right)\\ \text{diag}\left(\frac{n_{11}n^2}{2\phi^2}, 1\right)\end{cases} \quad \text{for} \quad K = \begin{cases}K_{21}\\ K_{111} \;\&\; K_{112}\end{cases}$$

- For $c_T = 1$ and $\phi \gg 1$ or $c_T \gg 1$ and $\phi < 1$, $\widehat{V}_{\rm I}$ Develops A Plateau With Almost Constant Potential Energy Density, If

$$2n = \begin{cases}n(n_{21}-1)\\ nn_{11}\end{cases} \quad \Rightarrow \quad \begin{cases}n_{21} = 3\\ n_{11} = 2\end{cases} \quad \text{for} \quad K = \begin{cases}K_{21}\\ K_{111} \;\text{and}\; K_{112}\end{cases} \quad (\text{Recall that } f \sim \phi^n)$$

This **Empirical Criterion** Is Very Precise Since The Data on $n_{\rm s}$ Allows Only Tiny (Of Order 0.001) Deviations[4].

- The η Problem Within SUGRA is Resolved by Requiring $c_T \gg 1$ and N (These Signal A **Mild Tuning**).

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- Along the Inflationary path, $\widehat{V} = \widehat{V}_{\rm I}$ and the Corresponding Hubble Parameter $\widehat{H}_{\rm I}$ Become (Almost Constant):

$$\widehat{V}_{\rm I} = \frac{\lambda^2 f_\phi^2}{4c_T^4\phi^{2n}} \quad \text{and} \quad \widehat{H}_{\rm I} = \frac{\widehat{V}_{\rm I}^{1/2}}{\sqrt{3}} = \frac{\lambda f_\phi}{2\sqrt{3}c_T^2\phi^n}, \quad \text{With} \quad f_\phi = 2^{n/2-1} - c_T\phi^n < 0.$$

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Stability of The Inflationary Direction and Kähler Potentials

- To Consolidate the Validity of Our Inflationary Scenario We have to Check the **Stability** of the Inflationary Trajectory $s = \bar{s} = \theta = 0$ w.r.t the Fluctuations Of The Various Fields, i.e.

$$\left.\frac{\partial V}{\partial \widehat{\chi}^{\alpha}}\right|_{s=\bar{s}=\theta=0} = 0 \quad \text{and} \quad \widehat{m}^2_{\chi^{\alpha}} > 0 \quad \text{Where} \quad \widehat{m}^2_{\chi^{\alpha}} = \text{Egv}\left[\widehat{M}^2_{\alpha\beta}\right] \quad \text{With} \quad \widehat{M}^2_{\alpha\beta} = \left.\frac{\partial^2 V}{\partial \widehat{\chi}^{\alpha}\,\partial \widehat{\chi}^{\beta}}\right|_{\theta=s=\bar{s}=0} \quad \text{and} \quad \chi^{\alpha} = \theta, s, \bar{s}.$$

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Scalar **Mass-Squared Spectrum** for $K = K_{111}, K_{21}$ and K_{112} Along The Inflationary Trajectory

Fields	Eingestates	Masses Squared			
			$K = K_{21}$	$K = K_{111}$	$K = K_{112}$
1 Real Scalar	$\widehat{\theta}$	$\widehat{m}^2_\theta/\widehat{H}^2_{\rm I}$	$4(2^{n-2} - c_T\phi^n f_\phi)/f^2_\phi$	$6(2^{n-2} - c_T\phi^n f_\phi)/f^2_\phi$	
1 Complex Scalar	$\widehat{s}, \widehat{\bar{s}}$	$\widehat{m}^2_s/\widehat{H}^2_{\rm I}$	$(2^n/f^2_\phi$ **-2**$)$	$3\cdot 2^{n-1}/f^2_\phi$	**$6(1/n_2$** $+2^{n-2}/f^2_\phi)$
2 Weyl Spinors	$\widehat{\psi}_\pm$	$\widehat{m}^2_{\psi_\pm}$	$2^{n-2}\lambda^2/3c^4_T\phi^{2n}$	$2^{n-3}\lambda^2/c^4_T\phi^{2n}$	

- For $K = K_{21}$, We obtain $\widehat{m}^2_S < 0$. This can be Rectified Including **Higher Order terms** of the Form $k_S|S|^4$ with $k_S \sim 1$, But Then The Symmetry of Kähler Manifold is Lost. Also, the Predictability Of The Theory Is Lost Since Terms $k_{S\Phi}|S|^2|\Phi|^2$ Change[5] The Resulting n_s and r. **Alternatively** We may Assume that $S^2 = 0$.

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- For $K = K_{112}$ with $n_2 \leq 6$ We Obtain $\widehat{m}^2_s > \widehat{H}^2_{\rm I}$ And So, the **Standard** 1-field Inflationary Formalism can be Applied.

[5] *C.P. (2014).*

The Kähler Manifold Corresponding to K_{112}

- The Structure of the $SU(1,1)/U(1)$ Coset Space Becomes More Transparent if We Define [6]

$$f(T) = \frac{1}{2}\frac{1 - Z/\sqrt{n_{11}}}{1 + Z/\sqrt{n_{11}}} \, .$$

We Can Show That The Theory Based on K_{112} and W is Equivalent With this Based on $\widetilde{K}_{112}$ and $\widetilde{W}$

$$\widetilde{K}_{112} = -n_{11}\ln\left(1 - \frac{|Z|^2}{n_{11}}\right) + n_2 \ln\left(1 + \frac{|S|^2}{n_2}\right) \quad \text{and} \quad \widetilde{W} = W(1 + Z/\sqrt{n_{11}})^{n_{11}}$$

since the **Generalized Kähler Potential** $\mathcal{G} = K + \ln|W|$ Remains **Unchanged**.

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- These Transformations Can Be Used To Define Transitive Actions Of The 2×2 Matrices

$$U_1 = \begin{pmatrix} a_1 & b_1 \\ b_1^* & a_1^* \end{pmatrix} \quad \text{and} \quad U_2 = \begin{pmatrix} a_2 & b_2 \\ -b_2^* & a_2^* \end{pmatrix}$$

on the Scalar Field Manifolds Parameterized by Z and S. These Matrices Have The properties

$$U_1^\dagger \sigma_3 U_1 = \sigma_3 \quad \text{and} \quad U_2^\dagger U_2 = I \quad \text{with} \quad \sigma_3 = \text{diag}\,(1,-1) \quad \text{and} \quad I = \text{diag}\,(1,1) .$$

Therefore, They Provide Representations Of The $SU(1,1)$ and $SU(2)$ Groups Respectively.

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$$U_1 = \begin{pmatrix} a_1 & b_1 \\ b_1^* & a_1^* \end{pmatrix} \quad \text{and} \quad U_2 = \begin{pmatrix} a_2 & b_2 \\ -b_2^* & a_2^* \end{pmatrix}$$

on the Scalar Field Manifolds Parameterized by Z and S. These Matrices Have The properties

$$U_1^\dagger \sigma_3 U_1 = \sigma_3 \quad \text{and} \quad U_2^\dagger U_2 = I \quad \text{with} \quad \sigma_3 = \text{diag}\,(1,-1) \quad \text{and} \quad I = \text{diag}\,(1,1) .$$

Therefore, They Provide Representations Of The $SU(1,1)$ and $SU(2)$ Groups Respectively.

- As Regards The **Scalar Curvatures** Of The Spaces, We Apply The Formula

$$R_K = g^{-3}\left(g_{,z}g_{,z^*} - g g_{,zz^*}\right) = \begin{cases} -2/n_{11}, & \text{for } SU(1,1)/U(1) \\ 2/n_2, & \text{for } SU(2)/U(1) \end{cases} \quad \text{With } g = \widetilde{K}_{112} \text{ and } z = Z \text{ or } z = S, \text{ Respectively.}$$

[6] *J.R. Ellis, C. Kounnas and D.V. Nanopoulos (1984); J. Ellis, D.V. Nanopoulos and K.A. Olive (2013).*

Testing Against Observations

- The **Slow-Roll Parameters** Are Determined As Follows:

$$\widehat{\epsilon} = \frac{2^{n-2}}{f_\phi^2} \quad \text{and} \quad \widehat{\eta} = \frac{2^{n/2}}{f_\phi^2}\left(2^{n/2} - c_T \phi^n\right) \quad \text{With} \quad J = \frac{n}{\phi}$$

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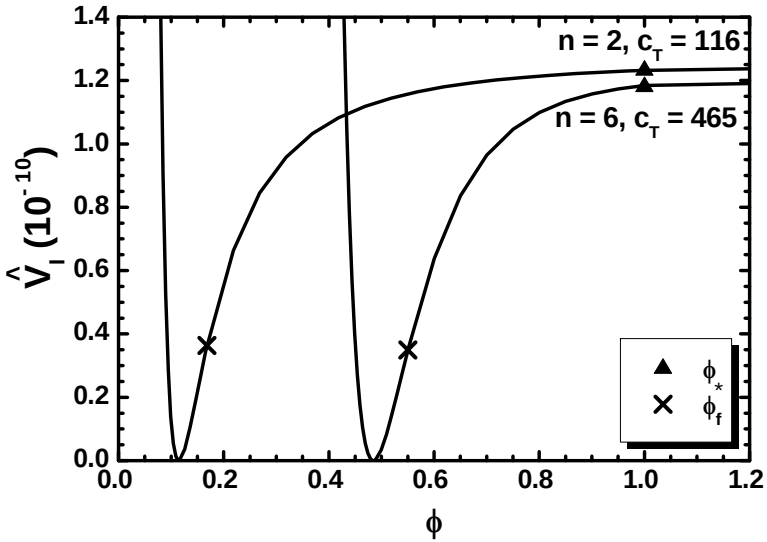
$$\phi_{\rm f} = \max\left\{\sqrt{2}\left(\frac{1+\sqrt{2}}{2c_T}\right)^{1/n}, \sqrt{2}\left(\frac{\sqrt{3}}{2c_T}\right)^{1/n}\right\}.$$

- We can then Calculate $\widehat{N}_\star$ and from it $\phi_\star$, as Follows:



$$\widehat{N}_\star \simeq \frac{c_T}{2^{n/2}}\left(\phi_\star^n - \phi_{\rm f}^n\right) - \frac{n}{2}\ln\frac{\phi_\star}{\phi_{\rm f}} \quad \Rightarrow \quad \phi_\star \simeq \sqrt{2}\left(\widehat{N}_\star/c_T\right)^{1/n}.$$

- There is a Lower Bound on c_T, Above Which $\phi_\star < 1$ – and so Terms Φ^l with $l > n$ Are Harmless. Indeed,

$$\phi_\star \leq 1 \quad \Rightarrow \quad c_T \geq 2^{n/2}\widehat{N}_\star \simeq 110, \quad \text{with} \quad \widehat{N}_\star = 55 \quad \text{and} \quad n = 2.$$

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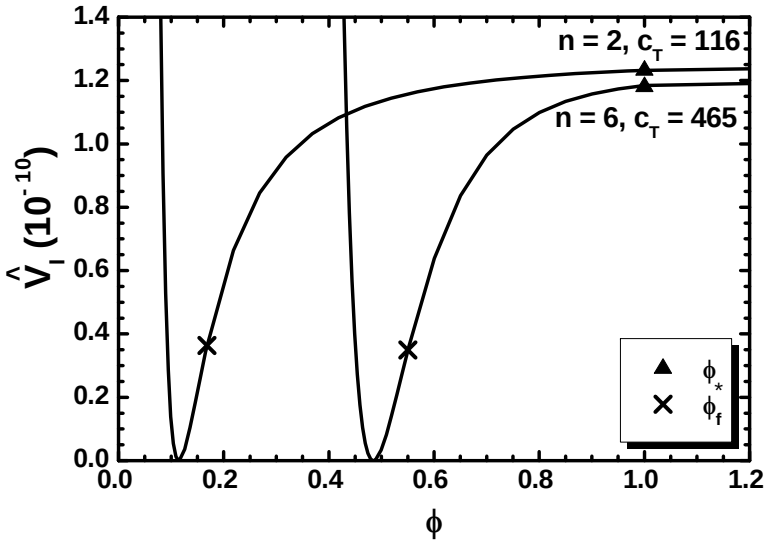

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- The **Power Spectrum Normalization** Implies A **Dependence of λ on c_T** Independently of n

$$A_{\rm s}^{1/2} = \frac{\lambda(1-2\widehat{N}_\star)^2}{16\sqrt{3}c_T\pi\widehat{N}_\star} \simeq 4.627\cdot 10^{-5} \quad \Rightarrow \quad \lambda \simeq 4\pi\sqrt{3A_{\rm s}}c_T/\widehat{N}_\star \Rightarrow c_T \simeq 41637\lambda\,.$$

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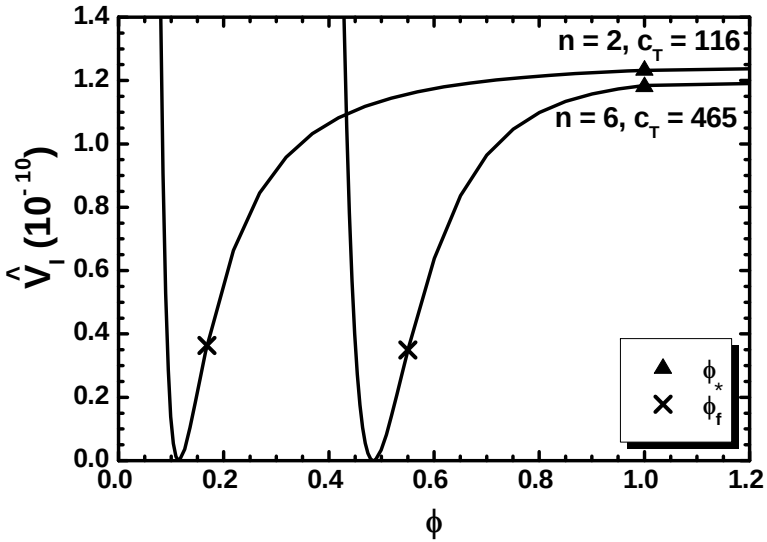

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$$n_{\rm s} = \frac{4(\widehat{N}_\star-3)\widehat{N}_\star - 3}{(1-2\widehat{N}_\star)^2} \simeq 1 - \frac{2}{\widehat{N}_\star} \simeq 0.964, \quad \alpha_{\rm s} = -\frac{16\widehat{N}_\star(3+2\widehat{N}_\star)}{(1-2\widehat{N}_\star)^3} \simeq \frac{-2}{\widehat{N}_\star^2} \simeq -6.3\cdot 10^{-4}, \quad r = \frac{32}{(1-2\widehat{N}_\star)^2} \simeq \frac{8}{\widehat{N}_\star^2} \simeq 0.0024.$$

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