Brownian Motion in the Landscape

Francisco Gil Pedro

in collaboration with:
Alexander Westphal

String Pheno 2016, Ioanina

Motivation

String theory as …

Many choices in deriving 4D physics:

Compact Manifold

Non-perturb. eff.

Fluxes

Brane configuration

…

Looking for vacua ⟷ Scanning over vast parameter space

Limitations of EFT: weak coupling/ large volume

Forced to look under the lamppost

Motivation

Statistical description of the landscape

Random Matrix Theory

Applied in other branches of physics:

- Nuclear physics
- Condensed matter

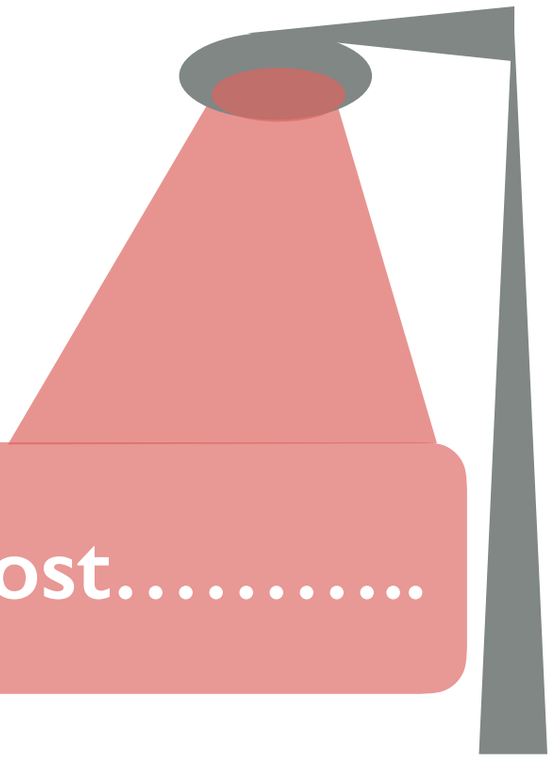
Motivation

Statistical description of the landscape

Random Matrix Theory

Applied in other branches of physics:

- Nuclear physics
- Condensed matter



Apply RMT to look away from the lamp post……..

c.f. Denef, Douglas, McAllister, Rummel, Shui, Sumitomo, Marsh, Pajer, Baumann, Bachlechner, McGuirk, Long, Stout, Tye, Chen…

Wigner Landscape

Consider N scalar fields coupled to Einstein gravity

$$\mathcal{L} = \frac{1}{2}\partial\phi^a\partial\phi^a - \Lambda_v^4\sqrt{N}\left[v_0 + v_a\phi^a + \frac{1}{2}v_{ab}\phi^a\phi^b\right]$$

Wigner Landscape

Consider N scalar fields coupled to Einstein gravity

$$\mathcal{L} = \frac{1}{2}\partial\phi^a\partial\phi^a - \Lambda_v^4\sqrt{N}\left[v_0 + v_a\phi^a + \frac{1}{2}v_{ab}\phi^a\phi^b\right]$$

1 Let v_{ab} be a **Wigner matrix**

$$W = \frac{A + A^\dagger}{2}$$

$$A \sim \Omega(\mu, \sigma)$$

Wigner Landscape

Consider N scalar fields coupled to Einstein gravity

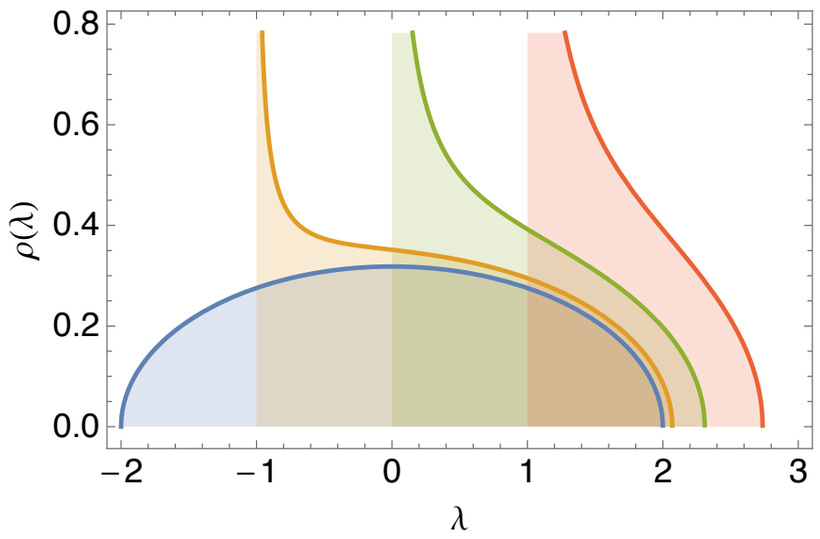
$$\mathcal{L} = \frac{1}{2}\partial\phi^a\partial\phi^a - \Lambda_v^4\sqrt{N}\left[v_0 + v_a\phi^a + \frac{1}{2}v_{ab}\phi^a\phi^b\right]$$

1 Let v_{ab} be a **Wigner matrix**

$$W = \frac{A + A^\dagger}{2}$$

$$A \sim \Omega(\mu, \sigma)$$

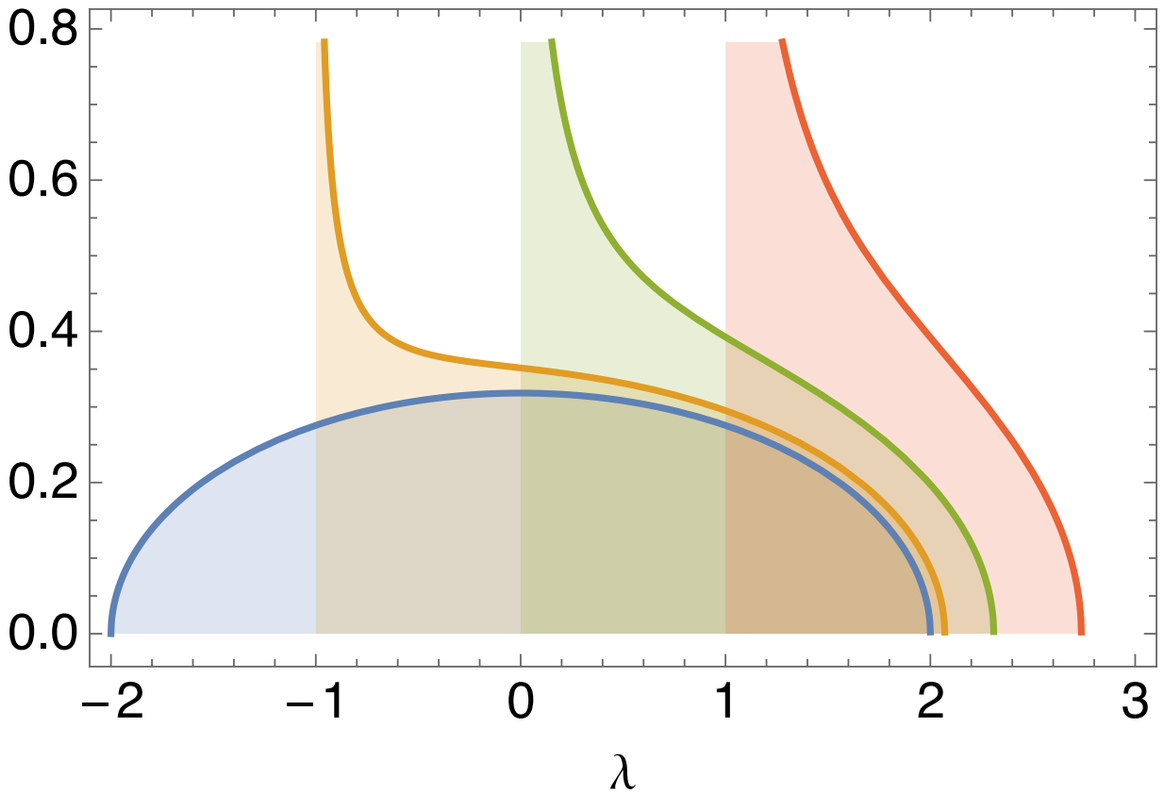
This defines the **Wigner Landscape**



- Equilibrium spectra with tachyons
- Positive definite fluctuated spectra

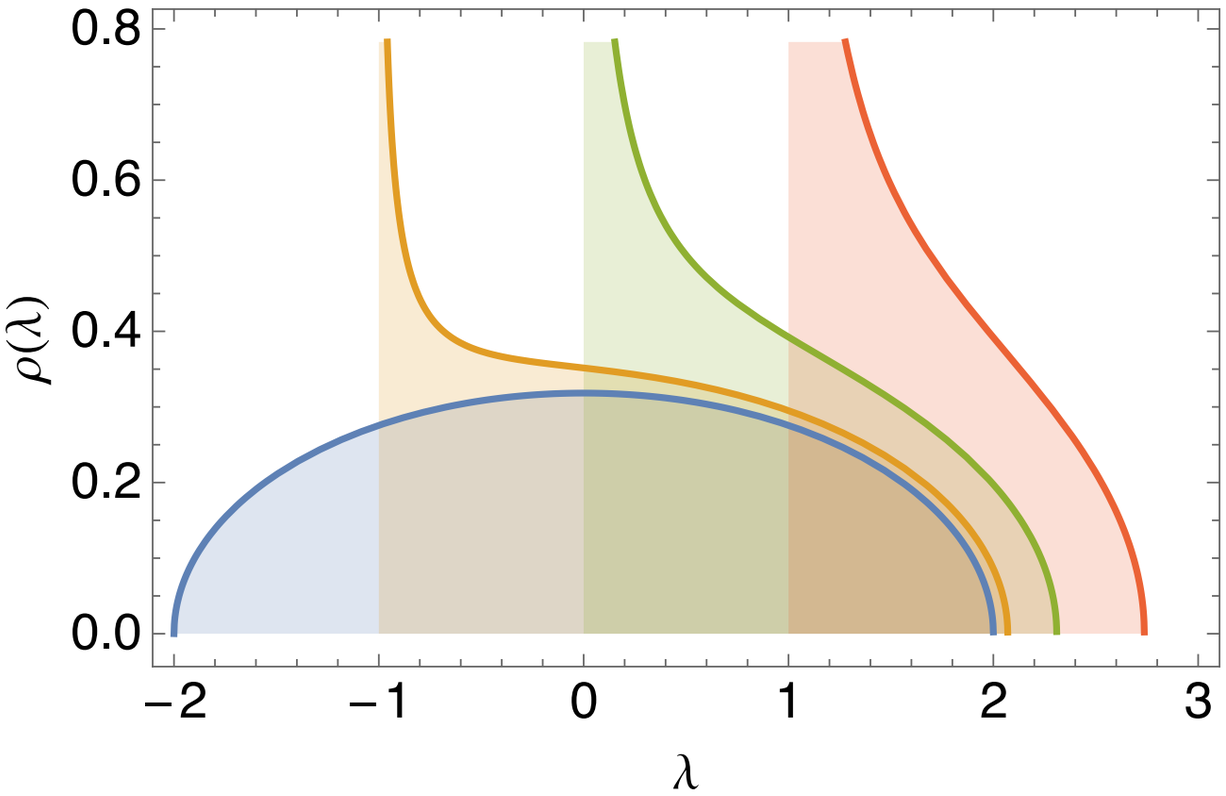
Wigner Landscape

Determine probabilities of occurrence of certain spectra



Wigner Landscape

Determine probabilities of occurrence of certain spectra



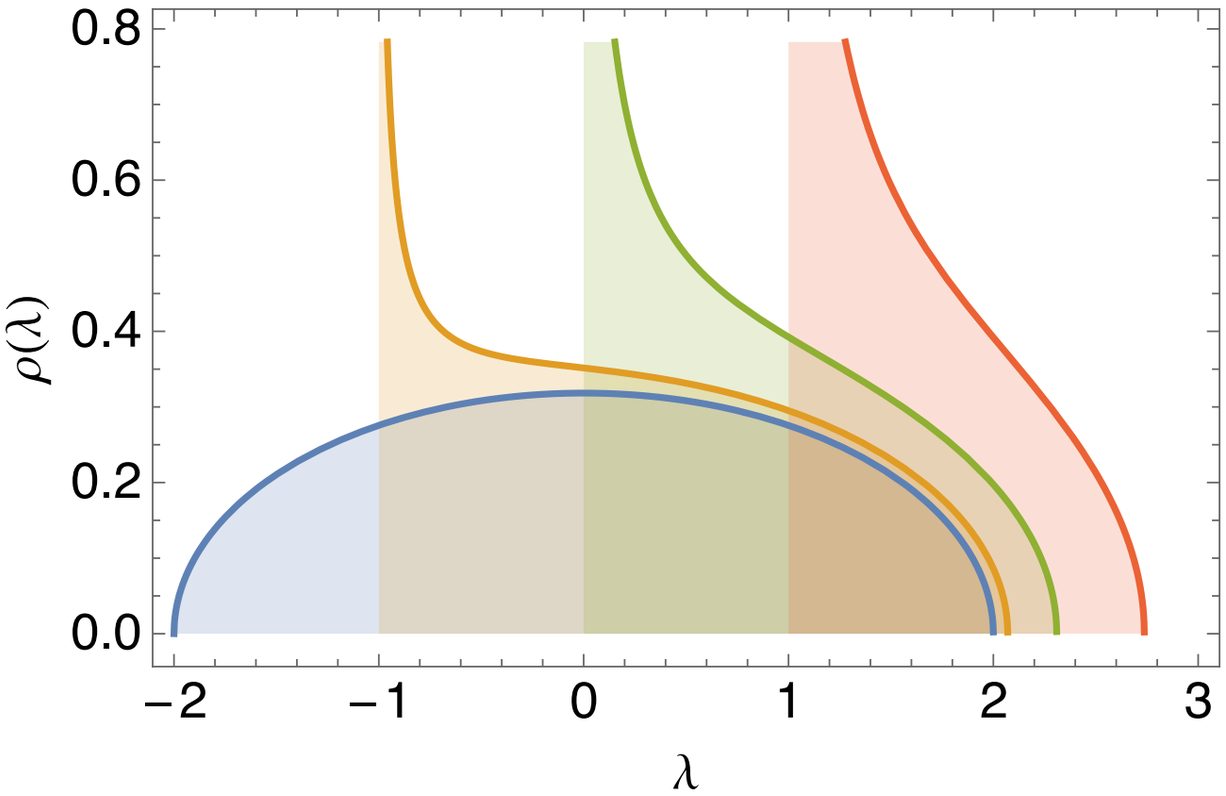
Use RMT for **Gaussian ensembles**

$$dP = \mathcal{C} \exp\left\{-\frac{\beta}{2\sigma^2} Tr[M^2]\right\} dM_{ij}$$

$$P(\forall\lambda > \zeta) = \int_{\zeta}^{+\infty} dP$$

Wigner Landscape

Determine probabilities of occurrence of certain spectra



Use RMT for **Gaussian ensembles**

$$dP = \mathcal{C} \exp\left\{-\frac{\beta}{2\sigma^2} Tr[M^2]\right\} dM_{ij}$$

$$P(\forall\lambda > \zeta) = \int_{\zeta}^{+\infty} dP$$

[Dyson]

1D gas of charged particles in equilibrium

$$H(\lambda) = -\frac{\beta}{2\sigma^2}\sum_{i=1}^{N}\lambda_i^2 + \frac{\beta}{2}\sum_{i\neq j}\log|\lambda_i - \lambda_j|$$

Wigner Landscape

Probabilities can be computed:

[Aazami&Easther: 2005]
[FGP&Westphal: 2013]

- numerically
- analytically via saddle point approximation

[Dean&Majumdar: 2006/8]

$$P(\forall\lambda > \zeta) = \exp\left(-\beta N^2 \Phi(\zeta)\right)$$

Wigner Landscape

Probabilities can be computed:

[Aazami&Easther: 2005]
[FGP&Westphal: 2013]

- numerically
- analytically via saddle point approximation

[Dean&Majumdar: 2006/8]

$$P(\forall\lambda > \zeta) = \exp\left(-\beta N^2 \Phi(\zeta)\right)$$

Rate function:

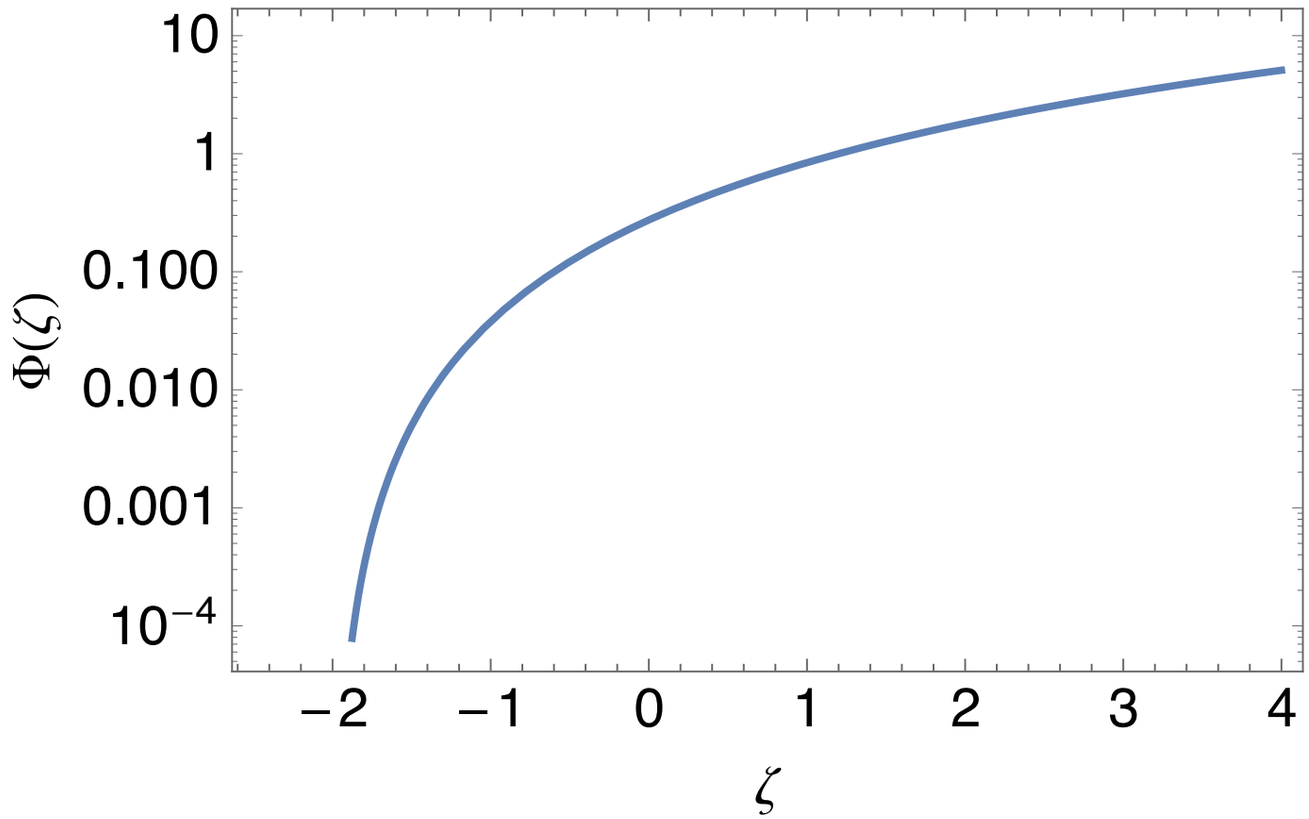
$$\Phi(\zeta) = \frac{1}{432}\left[72\zeta^2 - \zeta^4 + \left(30\zeta + \zeta^3\right)\sqrt{12+\zeta^2} + 108\ln 36 - 216\ln\left(-\zeta + \sqrt{12+\zeta^2}\right)\right]$$

Wigner Landscape

Probabilities can be computed:

- numerically [Aazami&Easther: 2005] [FGP&Westphal: 2013]
- analytically via saddle point approximation [Dean&Majumdar: 2006/8]

$$P(\forall \lambda > \zeta) = \exp\left(-\beta N^2 \Phi(\zeta)\right)$$



Wigner Landscape

What is more abundant in the landscape: minima or inflationary patches?

[FGP&Westphal:2013]

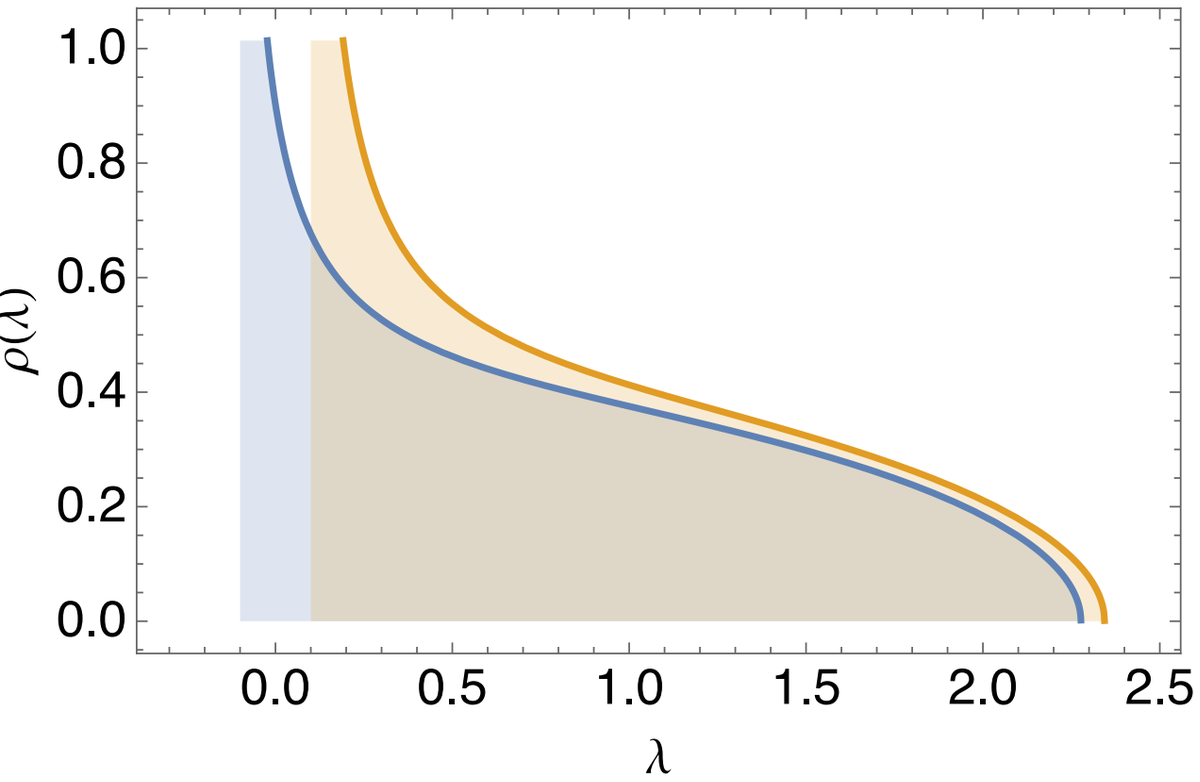
- $P(\text{min}) = P(\forall\lambda > \eta) = e^{-\beta N^2 \Phi(\eta)}$
- $P(\text{inf}) = P(\forall\lambda > -\eta) - P(\forall\lambda > \eta) = e^{-\beta N^2 \Phi(-\eta)} - e^{-\beta N^2 \Phi(\eta)}$

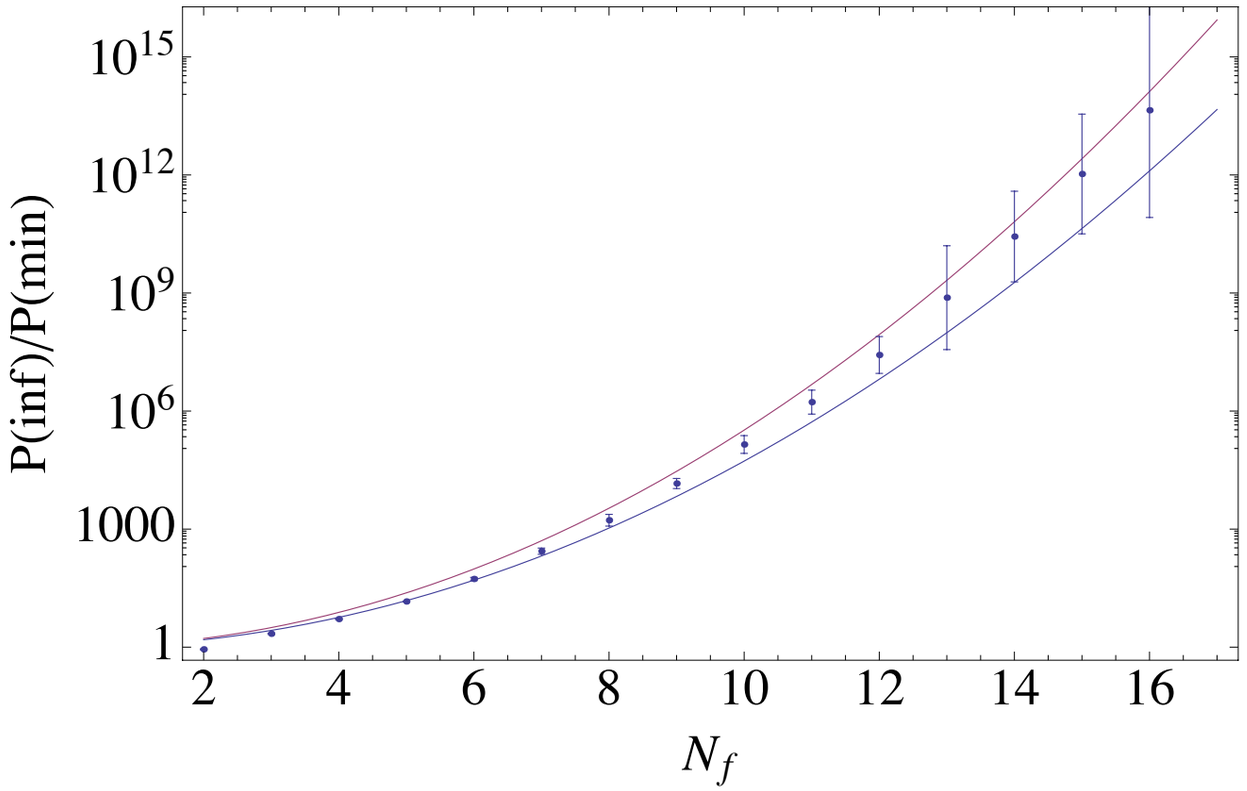
Wigner Landscape

[FGP&Westphal:2013]

$$\frac{P(\text{inf})}{P(\text{min})} \sim e^{\beta N^2 \Delta}$$

$$\Delta = \frac{4}{3\sqrt{3}}\eta + \mathcal{O}(\eta^2)$$



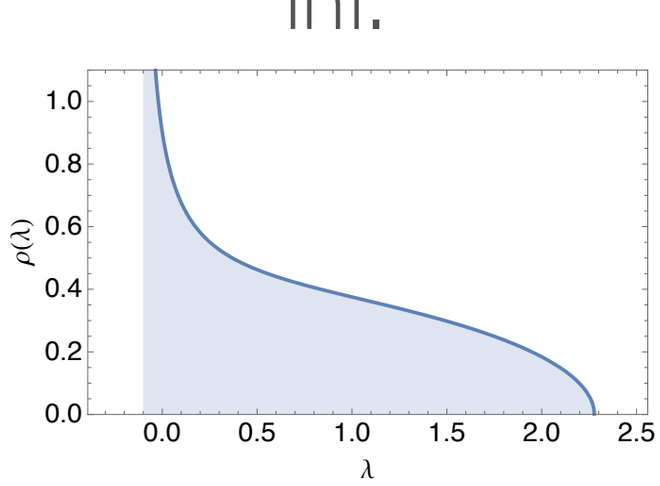


Wigner Landscape

Inflationary patches are **MUCH** more abundant than minima

How are they connected in the landscape?

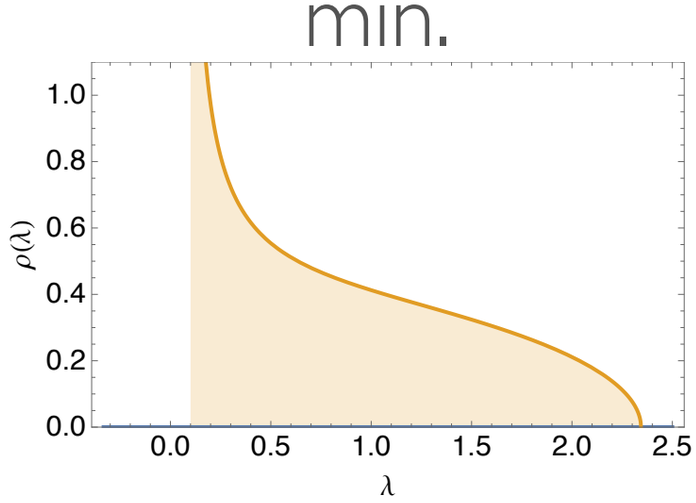
inf.



$\{\phi_i\}$

???

min.



$\{\phi_i + \delta\phi_i\}$

Want to find the **exit probability**

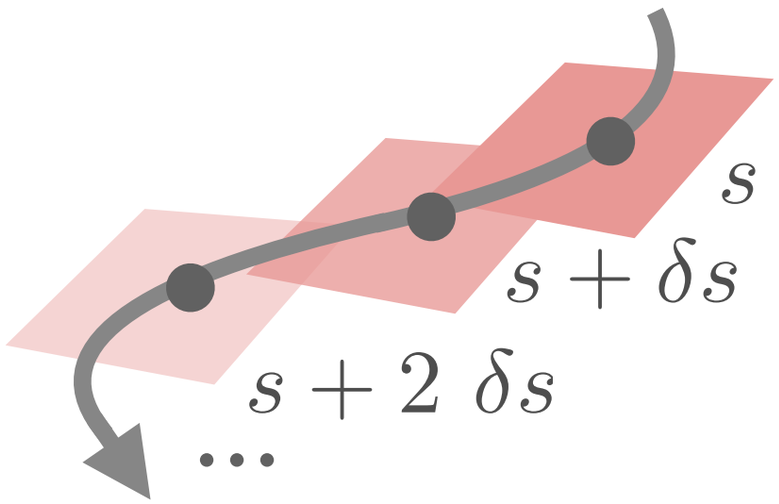
Wigner Landscape

Move away from the static ensemble

Wigner Landscape

Move away from the static ensemble

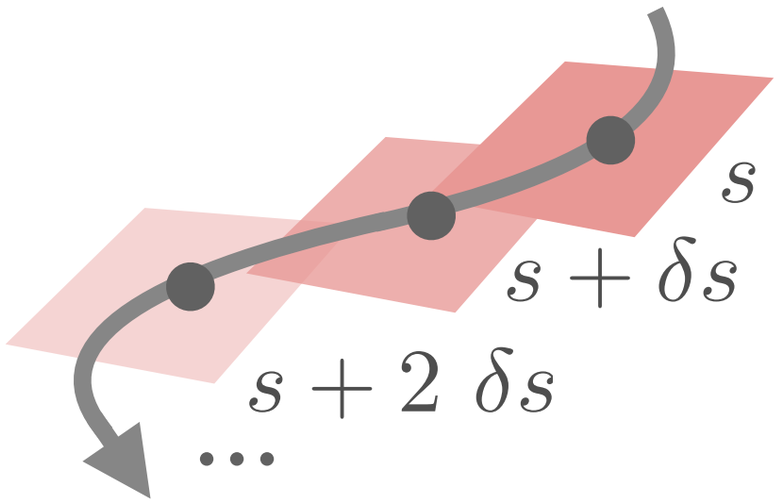
s coordinate along the background trajectory



Wigner Landscape

Move away from the static ensemble

s coordinate along the background trajectory



$$v_0(s+\delta s) = v_0(s) + v_a(s)\delta\phi^a$$

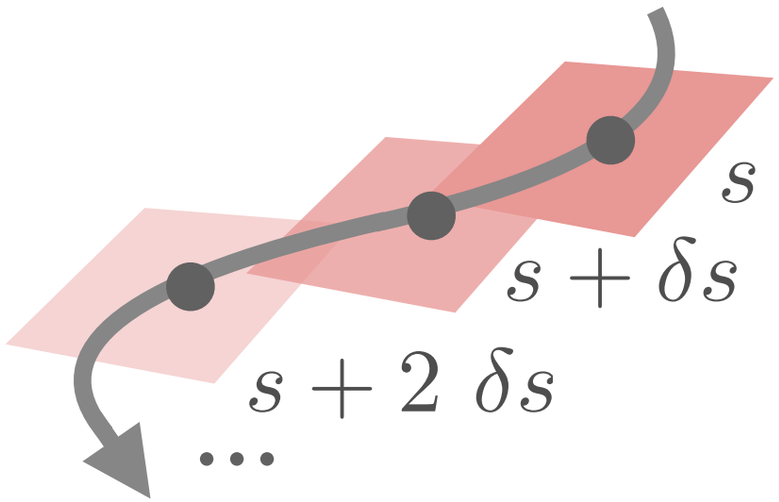
$$v_a(s+\delta s) = v_a(s) + v_{ab}(s)\delta\phi^b$$

$$v_{ab}(s+\delta s) = v_{ab}(s) + v_{abc}(s)\delta\phi^c$$

Wigner Landscape

Move away from the static ensemble

s coordinate along the background trajectory



$$v_0(s+\delta s) = v_0(s) + v_a(s)\delta\phi^a$$

$$v_a(s+\delta s) = v_a(s) + v_{ab}(s)\delta\phi^b$$

$$v_{ab}(s+\delta s) = v_{ab}(s) + v_{abc}(s)\delta\phi^c$$

"Time" dependence introduced by postulating **Brownian motion**

$$\langle \delta v_{ab}(s+\delta s)\rangle = -v_{ab}(s)\frac{|\delta\phi^a|}{\Lambda_h}$$

$$\langle \delta v_{ab}(s+\delta s)^2\rangle = -(1+\delta_{ab})\frac{|\delta\phi^a|}{\Lambda_h}\sigma^2$$

[Marsh et al:2013]

[Dias et al:2016]

Dyson Brownian Motion

[Dyson:1963]

Matrix Browian motion  Eigenvalue Browian motion

Time dependent 1D Coulomb gas

Applied to the string landscape by [Marsh et al:2013]

"time" ⟶ s field space distance along background trajectory

Locally reconstruct $V(\{\phi_i\})$ and study inflationary dynamics

[Dias et al:2016]

Dyson Brownian Motion

Sol. to Focker/Planck eq.:

[Dyson:1963]

$$P(M(s), M_0) = \int dP \qquad q \equiv e^{-\frac{s}{\sigma^2 f}}$$

$$dP = \mathcal{C} \ \exp\left\{-\frac{\beta}{2\sigma^2(1-q^2)}\mathrm{Tr}[(M - qM_0)^2]\right\} dM_{ij}$$

Dyson Brownian Motion

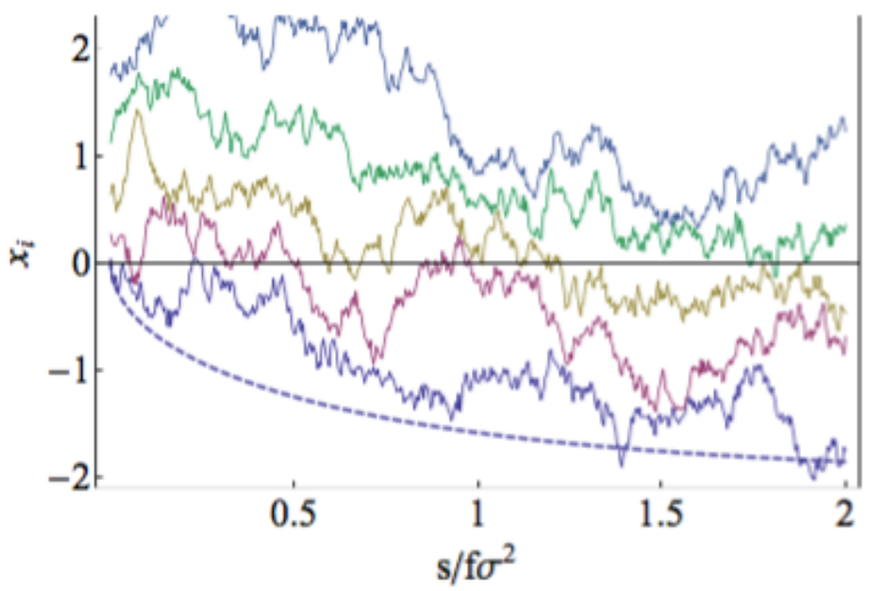
Sol. to Focker/Planck eq.: [Dyson:1963]

$$P(M(s), M_0) = \int dP \qquad q \equiv e^{-\frac{s}{\sigma^2 f}}$$

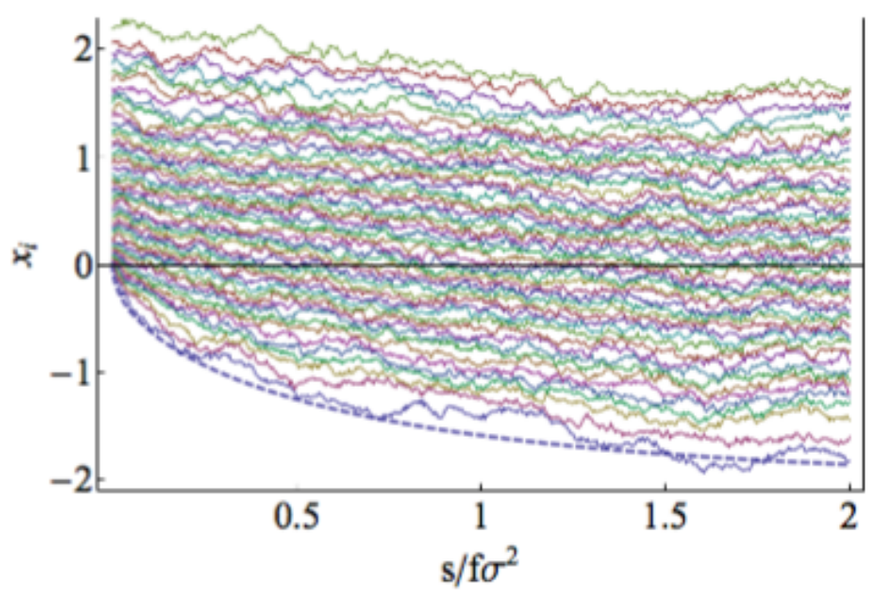
$$dP = \mathcal{C} \ \exp\left\{-\frac{\beta}{2\sigma^2(1-q^2)}\mathrm{Tr}[(M-qM_0)^2]\right\} dM_{ij}$$

Relaxation towards equilibrium configuration [Marsh et al:2013]

from Marsh et al: 2013



(a) $N = 5$



(b) $N = 50$

Dyson Brownian Motion

Estimate exit probability:

- Numerical evolution of matrices
- Analytical saddle point integration of dP

[Dean&Majumdar:2006/8] for time dependent case

[FGP&Westphal:2016]

Hamiltonian:

$$H(\lambda) = -\frac{\beta}{2}\sum_{i=1}^{N}\left\{\frac{\lambda_i^2}{\sigma^2(1-q^2)} - \frac{2q\lambda_i M_0^{ii}}{\sigma^2(1-q^2)}\right\} + \frac{\beta}{2}\sum_{i\neq j}\ln|\lambda_i - \lambda_j|$$

Each eigenvalue subject to **different** linear potential

To get a qualitative description **approximate**:

$$M_0^{ii} \to m \equiv \frac{1}{N}\sum_{i=1}^{N} M_0^{ii} = \frac{1}{N}\mathrm{Tr}[M_0] \equiv \langle\lambda_{M_0}\rangle.$$

mean eigenvalue of M_0

Dyson Brownian Motion

What is the probability of $M(s) : \forall\ \lambda > \zeta$

$$P(M(s), M_0) = \exp\left[-\beta N^2 \Psi(\zeta) + \mathcal{O}(N)\right]$$

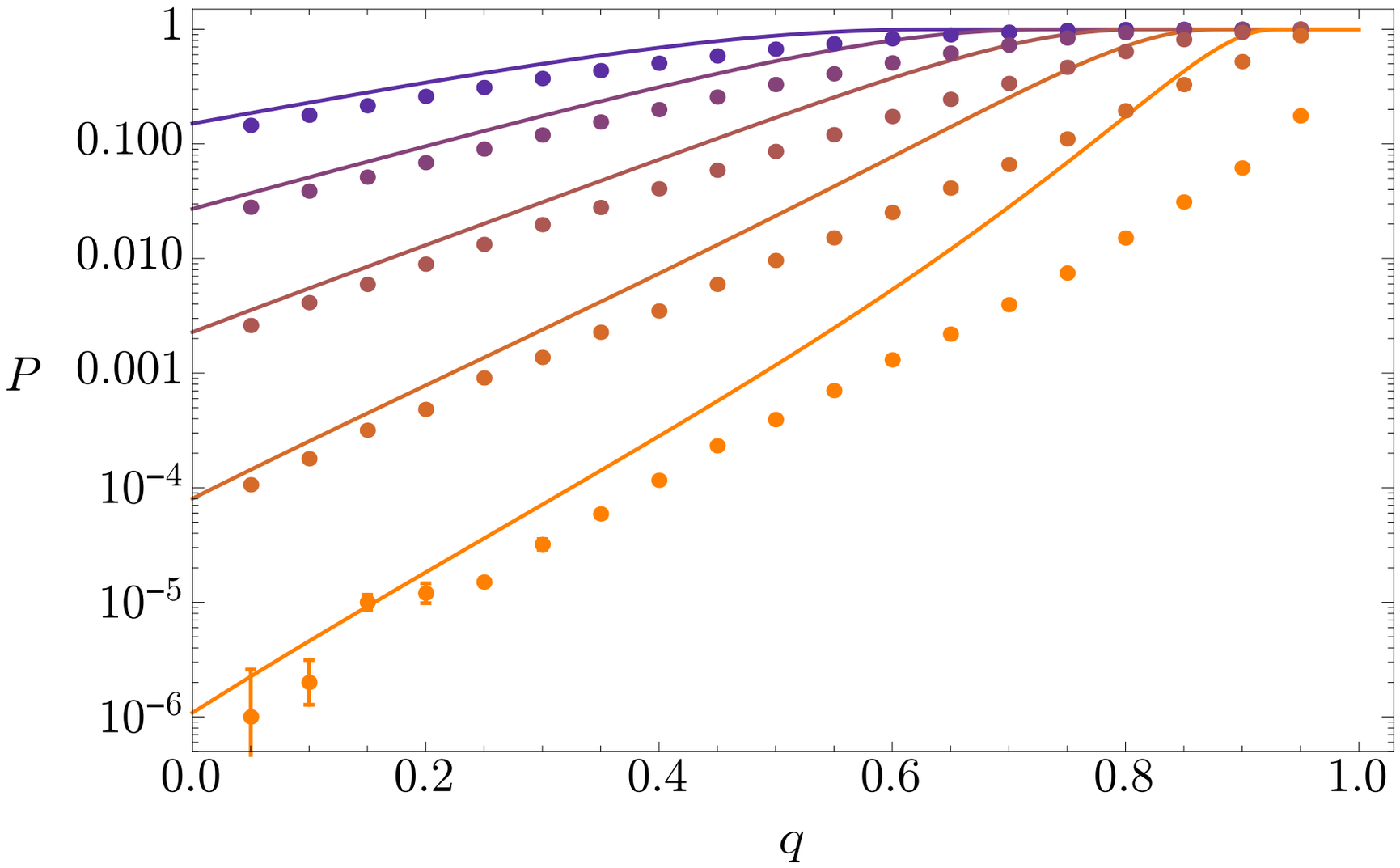
$$\Psi(\tilde{\zeta}) = \frac{1}{108\tilde{a}^2}\left\{36\tilde{a}\tilde{\zeta}^2 - \tilde{\zeta}^4 + (15\tilde{a}\tilde{\zeta} + \tilde{\zeta}^3)\sqrt{6\tilde{a} + \tilde{\zeta}^2} + 27\tilde{a}^2\left[\ln(72\tilde{a}) - 2\ln(2(\sqrt{6\tilde{a} - \tilde{\zeta}} - \tilde{\zeta}))\right]\right\}$$

$$\tilde{a} \equiv 2(1 - q^2)$$
$$b \equiv -2qm$$
$$\tilde{\zeta} \equiv \zeta + b/2$$

Dyson Brownian Motion

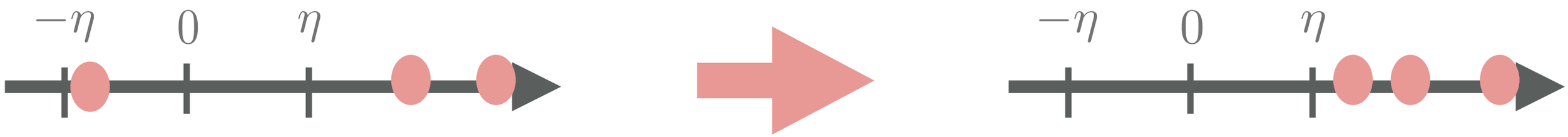
What is the probability of $M(s) : \forall\ \lambda > \zeta$

$$P(M(s), M_0) = \exp\left[-\beta N^2 \Psi(\zeta) + \mathcal{O}(N)\right]$$

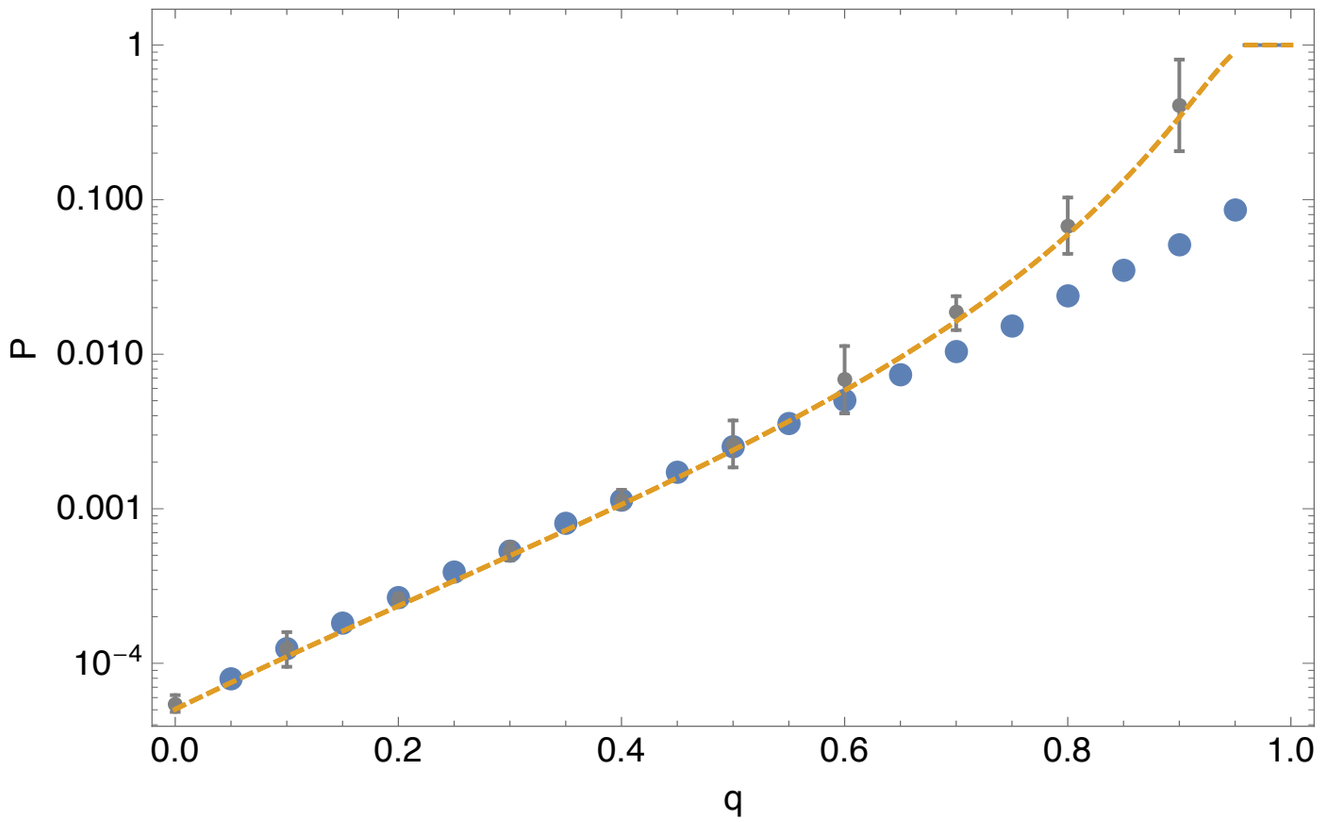


DBM in the Landscape

Apply method to **Wigner landscape**



inf



min

$\beta = 1$

$\eta = 1/10$

$N = 5$

Good agreement after one correlation length: $q < e^{-1} \sim 0.35$

$M_{inf} \gg \Lambda$: interested in small q regime

DBM in the Landscape

Transition probability is exponentially suppressed

? Who wins then ?

$$\frac{\mathcal{N}_{inf}}{\mathcal{N}_{min}} \quad \text{vs} \quad \frac{\mathcal{N}_{inf \wedge min}}{\mathcal{N}_{min}}$$

Conditional probability: $P(\mathrm{min}|\mathrm{inf}) = \dfrac{P(\mathrm{min} \cap \mathrm{inf})}{P(\mathrm{inf})}$

Anthropically relevant trajectories:

$$P(\mathrm{min} \cap \mathrm{inf}) = P(\mathrm{inf})\ P(\mathrm{min}|\mathrm{inf})$$

initial condition evolution

DBM in the Landscape

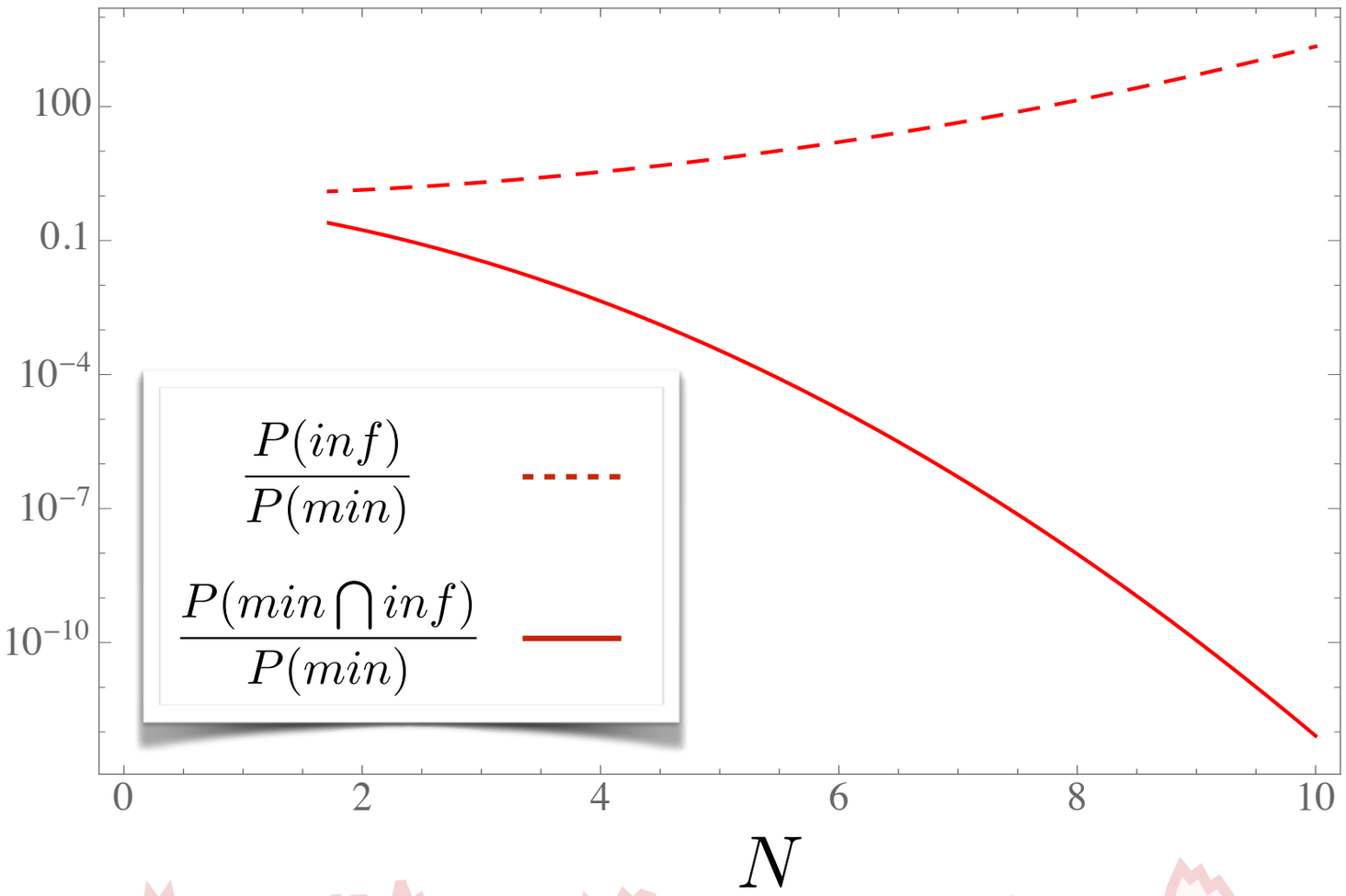
Globally $q \ll 1 \wedge |\eta| \ll 1$

$$P(\min \cap \inf) \simeq \exp\left[-\beta N^2 \frac{\ln 3}{2}\left(1 - \frac{16}{27\ln 3}q + \frac{8}{3\sqrt{3}\ln 3}\eta\right)\right]$$

DBM in the Landscape

Globally $q \ll 1 \wedge |\eta| \ll 1$

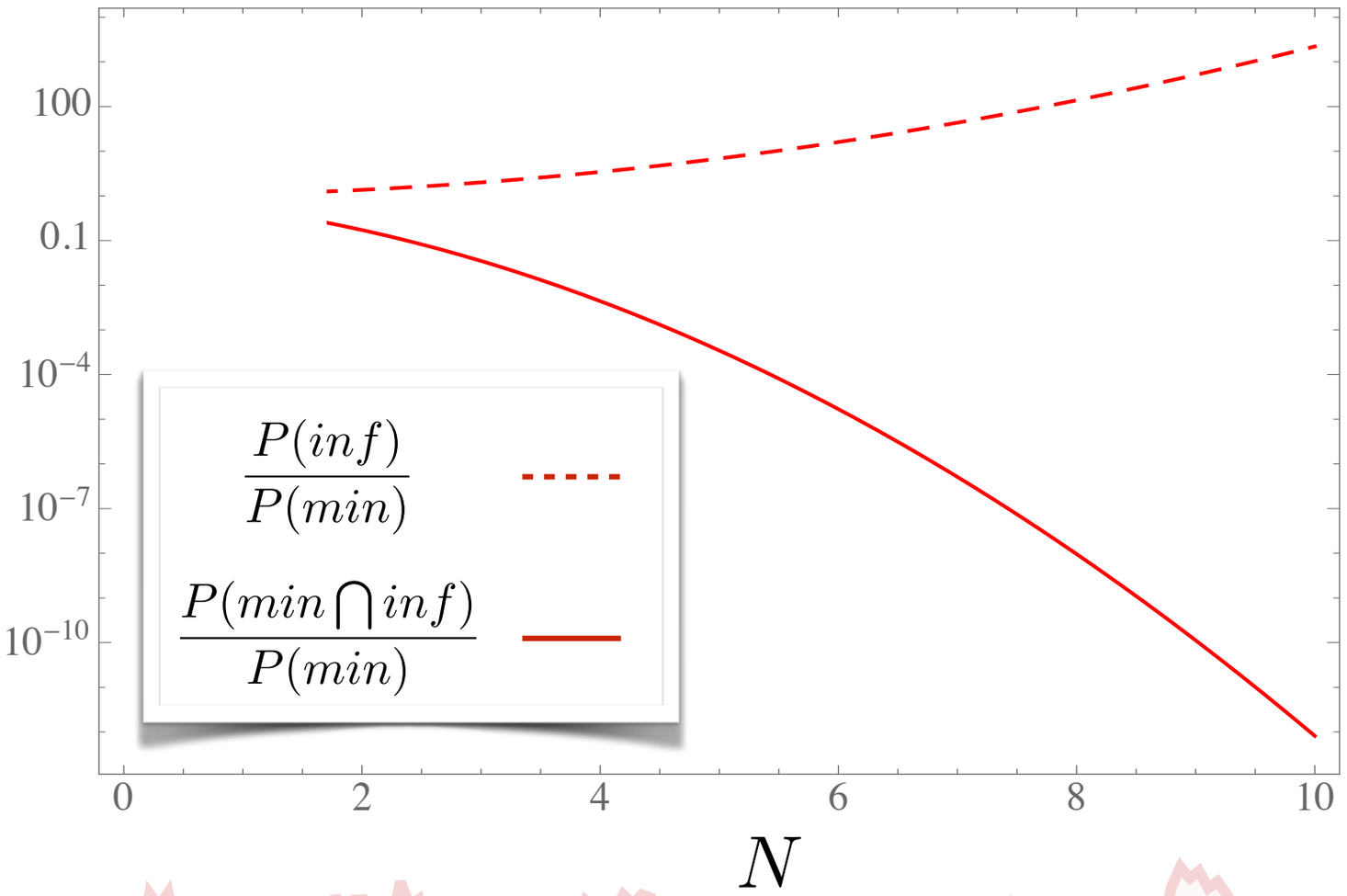
$$P(\min \cap \inf) \simeq \exp\left[-\beta N^2 \frac{\ln 3}{2}\left(1 - \frac{16}{27\ln 3} q + \frac{8}{3\sqrt{3}\ln 3}\eta\right)\right]$$



DBM in the Landscape

Globally $q \ll 1 \wedge |\eta| \ll 1$

$$P(\min \cap \inf) \simeq \exp\left[-\beta N^2 \frac{\ln 3}{2}\left(1 - \frac{16}{27\ln 3}q + \frac{8}{3\sqrt{3}\ln 3}\eta\right)\right]$$



Transition probability overcomes number count

Our History is highly unlikely !!!!

Summary

In a Wigner landscape:

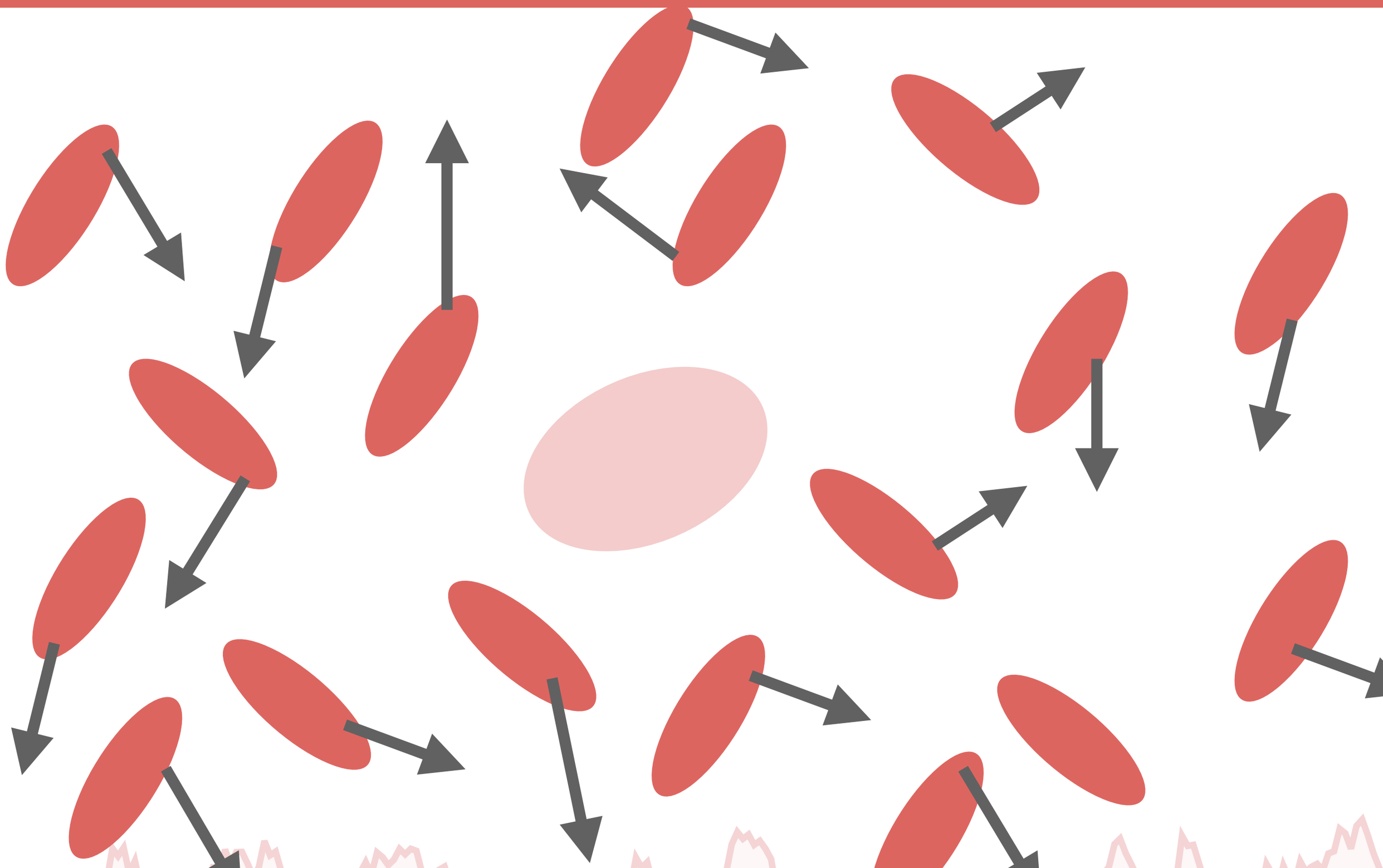
- Inflation/minima are very rare
- Inflation more abundant minima
- Successful trajectories are highly unlikely
- Exit probability overcomes number count
- Non-random structures are likely to dominate probabilities

Thank You

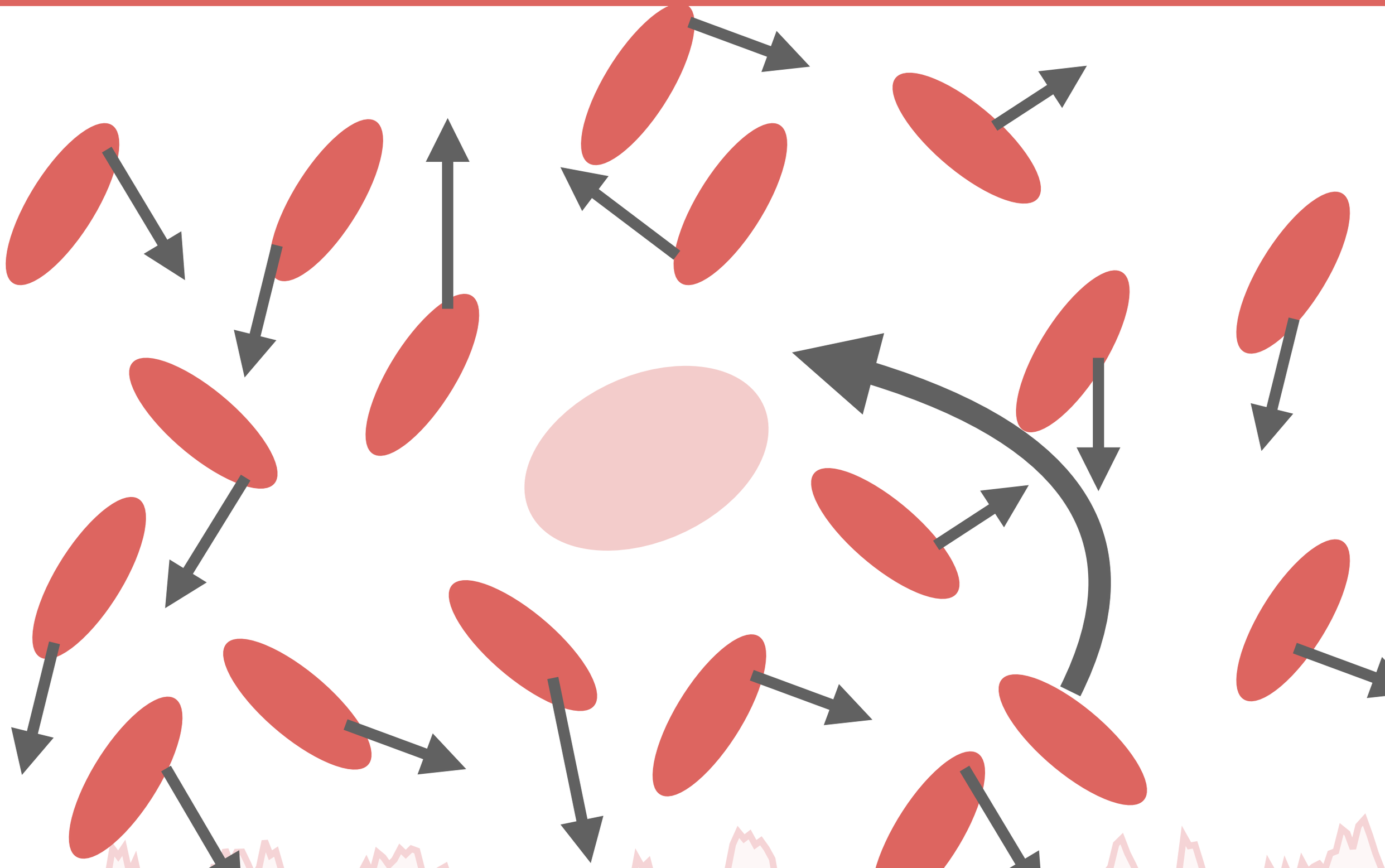
DBM in the Landscape

DBM in the Landscape

DBM in the Landscape



DBM in the Landscape



Wigner Landscape

A word on masses:

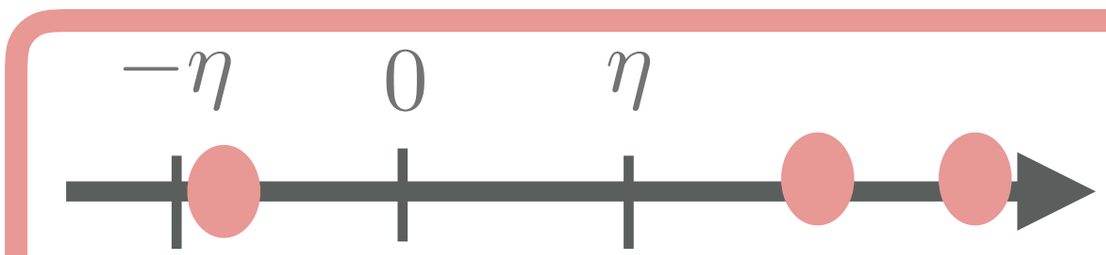
Typical mass scale set by σ

$$m^2 \in [-\sqrt{2N}\sigma, +\sqrt{2N}\sigma] \ \times \Lambda^2$$

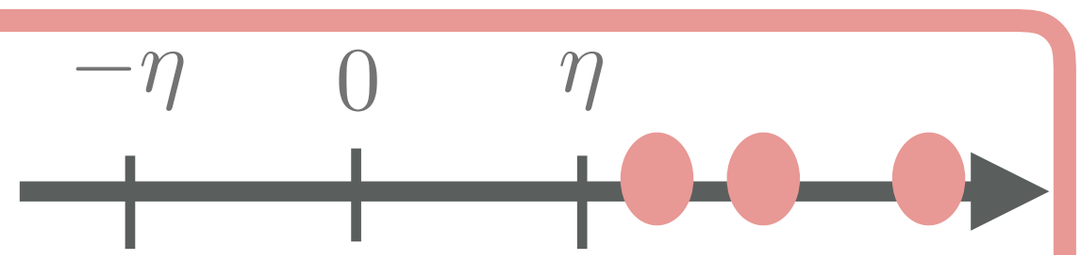
Assume $\Lambda \sim M_P$ and that $m \sim \mathcal{O}(M_P)$ → $\sigma \sim \dfrac{1}{\sqrt{N}}$

$$\eta = M_P^2 \frac{V''}{V} \ll 1 \qquad \text{choose} \quad \eta = 1/10$$

inf



min



DBM in the Landscape

Transition probability is exponentially suppressed

Conditional probability: $$P(\mathrm{min}|\mathrm{inf}) = \frac{P(\mathrm{min} \cap \mathrm{inf})}{P(\mathrm{inf})}$$

Anthropically relevant trajectories:

$$P(\mathrm{min} \cap \mathrm{inf}) = P(\mathrm{inf})\ P(\mathrm{min}|\mathrm{inf})$$

initial condition evolution

Globally $q \ll 1 \wedge |\eta| \ll 1$

$$P(\mathrm{min} \cap \mathrm{inf}) \simeq \exp\left[-\beta N^2 \frac{\ln 3}{2}\left(1 - \frac{16}{27 \ln 3} q + \frac{8}{3\sqrt{3}\ln 3}\eta\right)\right]$$

Wigner Landscape

Allows the use of RMT results for **Gaussian ensembles**

$$dP = \mathcal{C} \exp\left\{-\frac{\beta}{2\sigma^2} Tr[M^2]\right\} dM_{ij}$$

Compute probabilities by integration of the Gaussian jpdf

$$P(\forall \lambda > \zeta) = \int_{\zeta}^{+\infty} dP$$

Wigner Landscape

Allows the use of RMT results for **Gaussian ensembles**

$$dP = \mathcal{C} \exp\left\{-\frac{\beta}{2\sigma^2} Tr[M^2]\right\} dM_{ij}$$

Compute probabilities by integration of the Gaussian jpdf

$$P(\forall\lambda > \zeta) = \int_{\zeta}^{+\infty} dP$$

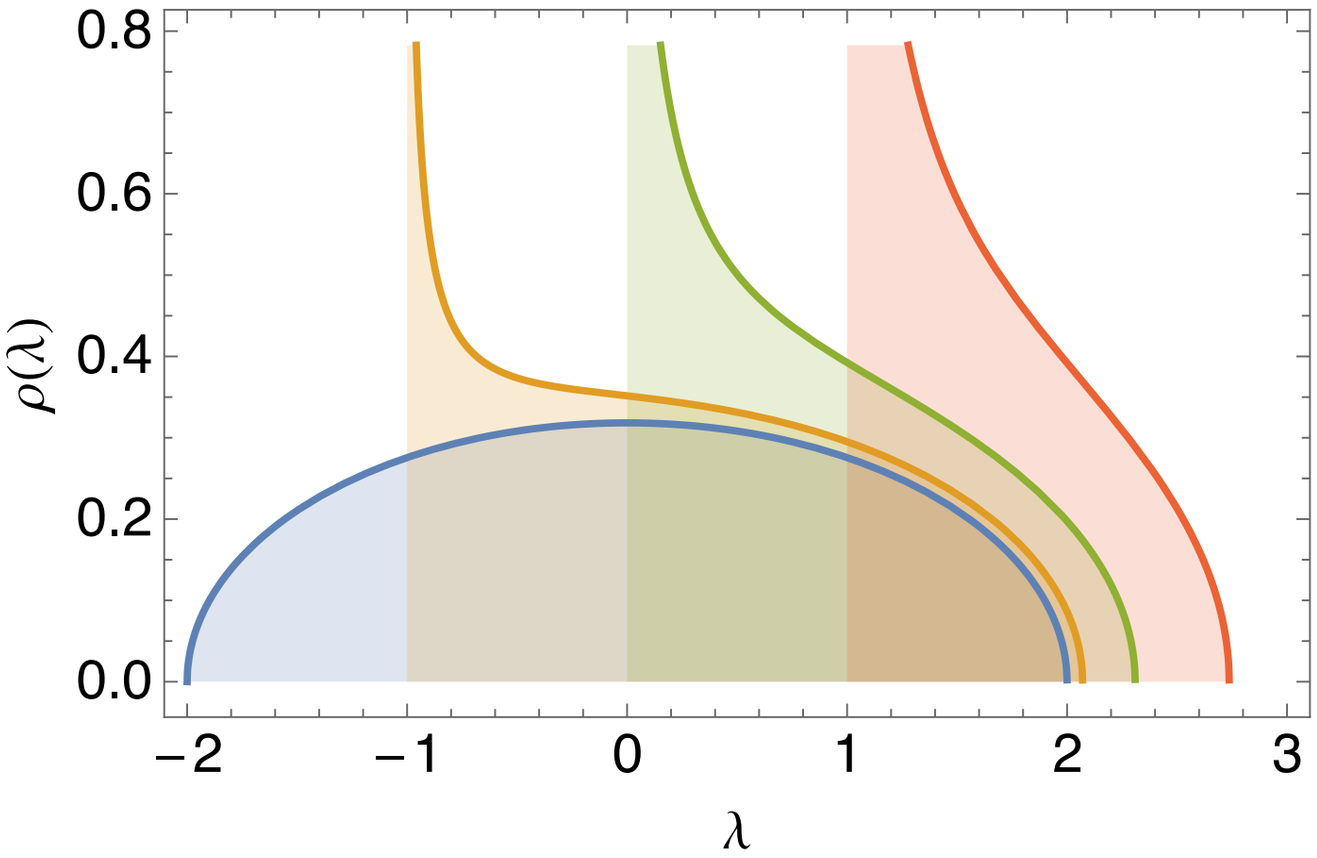
1D gas of charged particles in equilibrium [Dyson]

$$H(\lambda) = -\frac{\beta}{2\sigma^2}\sum_{i=1}^{N}\lambda_i^2 + \frac{\beta}{2}\sum_{i\neq j}\log|\lambda_i - \lambda_j|$$

Wigner Landscape

Can be computed:

- numerically [Aazami&Easther: 2005] [FGP&Westphal: 2013]
- analytically via saddle point approximation [Dean&Majumdar: 2006/8]



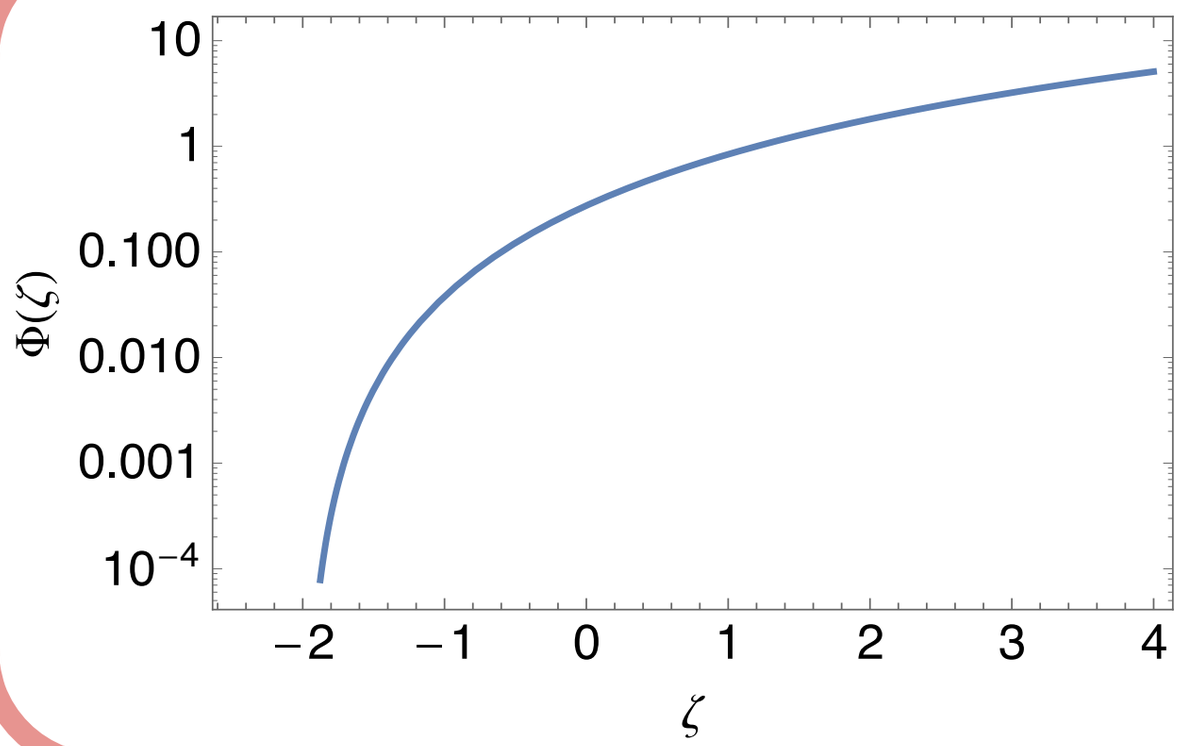
$$P(\forall\lambda > \zeta) = \exp\left(-\beta N^2 \Phi(\zeta)\right)$$

Wigner Landscape

[Dean and Majumdar: 2006/8]

Rate function:

$$\Phi(\zeta) = \frac{1}{432}\left[72\zeta^2 - \zeta^4 + \left(30\zeta + \zeta^3\right)\sqrt{12+\zeta^2} + 108\ln 36 - 216\ln\left(-\zeta + \sqrt{12+\zeta^2}\right)\right]$$



What is more abundant in the landscape: minima or inflationary patches?