String Pheno 2016 — Ιωάννινα

Constraining Axion Inflation With Gravitational Instantons

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with Arthur Hebecker, Patrick Mangat and Stefan Theisen

Intro

- Question whether inflation **large field** inflation is consistent with **Quantum Gravity / string theory** is currently undecided.

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2. Have no-go theorems based on the Weak Gravity Conjecture:

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     → see talk by Florent Baume

     → see however talk by Susha Parameswaran

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     → see talk by Miguel Montero for progress in this direction

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- **Here:** focus on axion inflation models with an effectively transplanckian period (via {kinetic} alignment…)

[Kim, Nilles, Peloso 04; Bachlechner, Dias, Frazer, McAllister 14; Hebecker, Mangat, Rompineve, LW 15; Shiu, Staessens, Ye 15; Blumenhagen, Herschmann, Wolf 16]

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→ see talks by Ralph Blumenhagen, Daniela Herschmann and Florian Wolf

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Towards a general no-go principle? **[Montero, Uranga, Valenzuela 15]**

- Ingredients of a axion inflation model: **axion field** + **gravity**
- Such Einstein-axion(-dilaton) theories admit **Euclidean solutions with finite action:** Gravitational Instantons

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- Such Einstein-axion(-dilaton) theories admit **Euclidean solutions with finite action:** Gravitational Instantons
- Expected to give rise to a contribution to the axion potential:

$$\delta V \sim e^{-n/f_{eff}} \cos\left(\frac{n\theta}{f_{eff}}\right) .$$

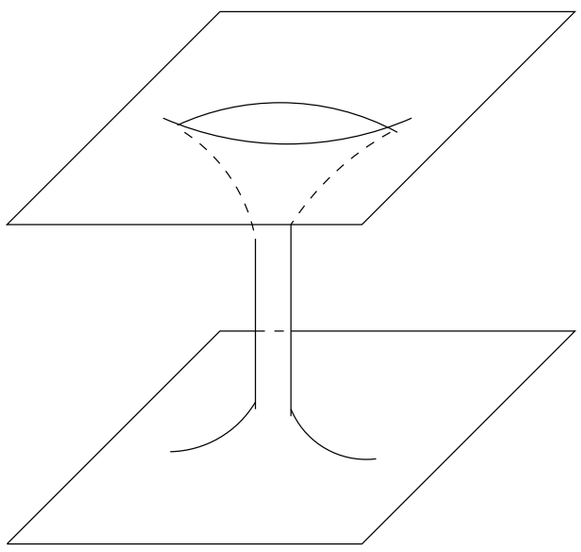
- For $f_{eff} > 1$ Gravitational Instantons contribute unsuppressed higher harmonics.
- Model-independent constraint for axion inflation?

Gravitational Instantons

- Euclidean solutions of 4d Einstein-axion(-dilaton) systems.
  [Giddings, Strominger 1988; Bergshoeff, Collinucci, Gran, Roest, Vandoren 2004]

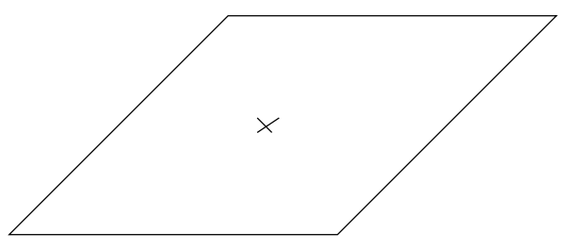
- Solution for metric: $ds^2 = \left(1 + \frac{C}{r^4}\right)^{-1} dr^2 + r^2 d\Omega_3^2$

$C < 0$



Wormhole

$C = 0$



Extremal Instanton

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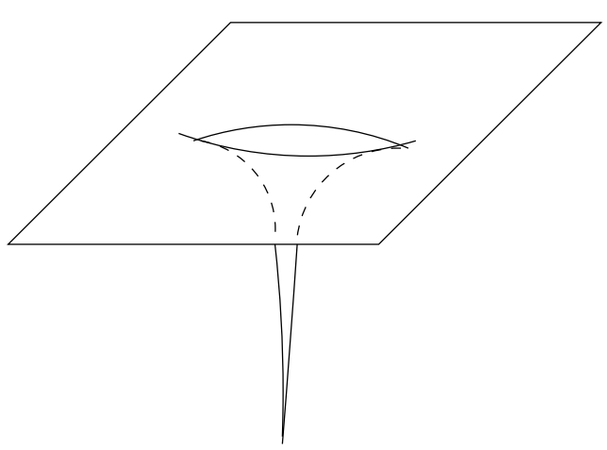

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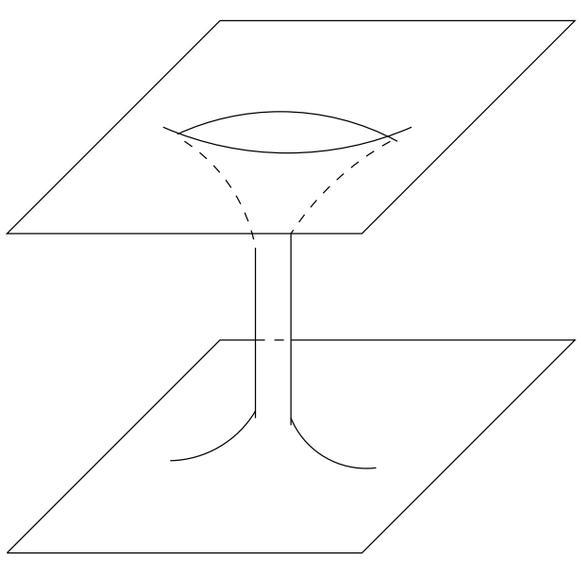

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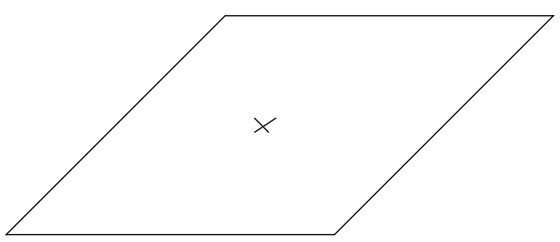

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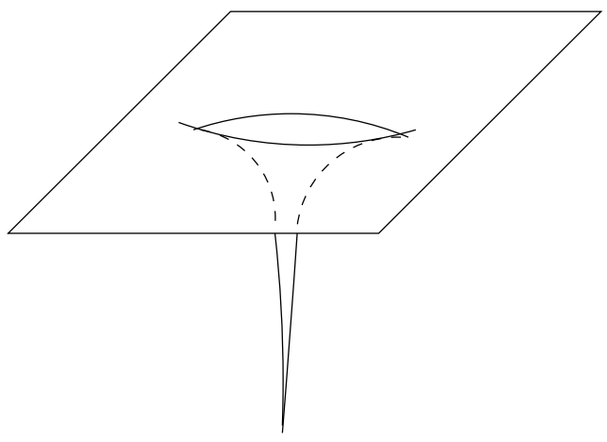

Cored Instanton

Do wormholes give rise to a potential of the form

$$\delta V \sim e^{-n/f_{eff}} \cos\left(\frac{n\theta}{f_{eff}}\right)$$

?

Wormhole Solution

- Metric ansatz:

$$ds^2 = \left(1 - \frac{L^4}{r^4}\right)^{-1} dr^2 + r^2 d\Omega_3^2$$

"Throat"

3-sphere S^3

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- Lagrangian: $\mathcal{L} \sim \mathcal{R} + f^2 |d\theta|^2 \longrightarrow \mathcal{L} \sim \mathcal{R} + \frac{1}{f^2} |dB_2|^2$

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$$\int_{S^3} H = n \in \mathbb{Z} \ .$$

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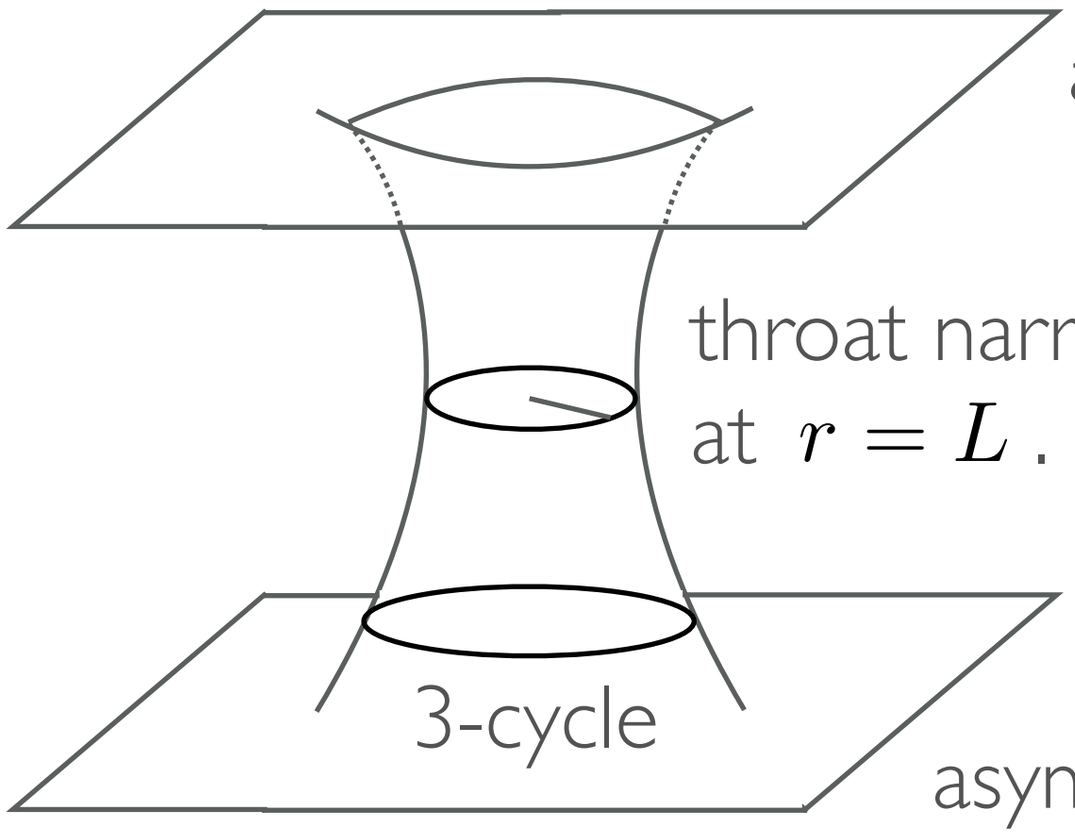
- The 'throat' exists due the compensation between the curvature term and the kinetic term of B_2.

Wormhole Solution

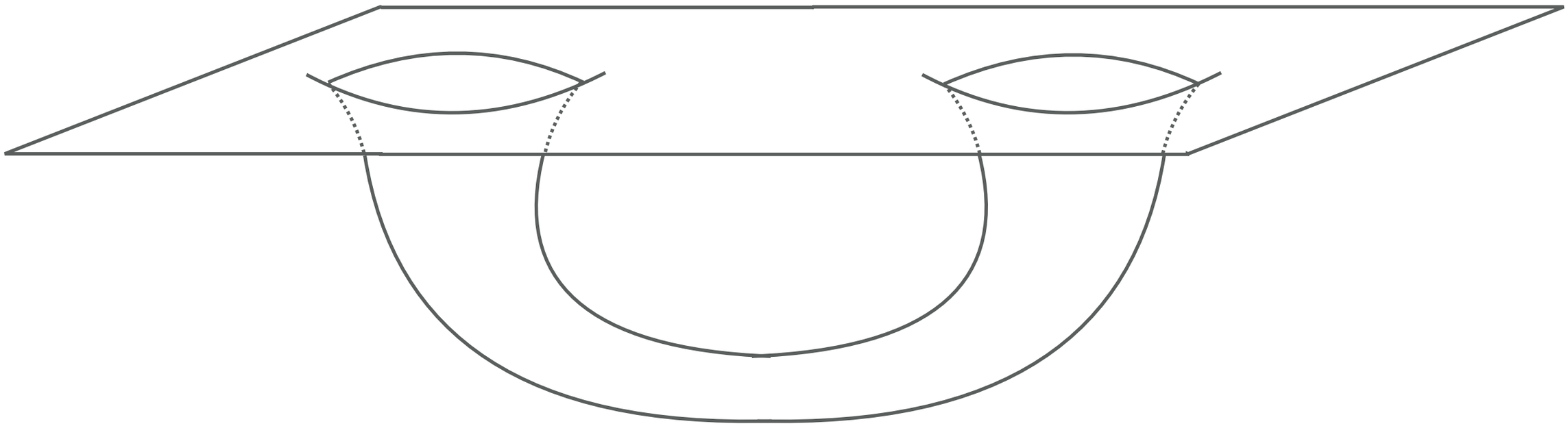
Metric solution:

$$ds^2 = \left(1 - \frac{L^4}{r^4}\right)^{-1} dr^2 + r^2 d\Omega_3^2 \quad \text{with} \quad L^4 = \frac{1}{24\pi^4}\frac{n^2}{f^2}$$



Wormhole Solution

Here: only consider wormholes that connect back to our space.



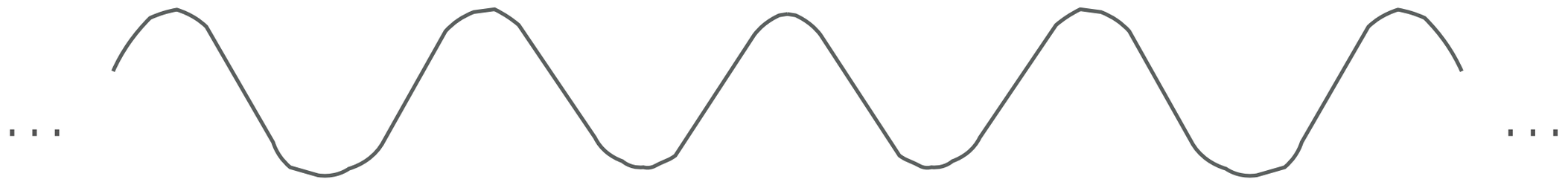
Do they give rise to a potential?

Review Coleman's calculation…

Instanton Potentials

Useful QM toy model: tunneling in a periodic potential:

$$H = \frac{p^2}{2} + V(x)$$



- Have set of minima labelled by integer n .
- Instanton: tunnelling $n \to n+1$, anti-instanton: $n \to n-1$
- Calculate transition amplitude $\langle n_+ | e^{-HT} | n_- \rangle$.
- Sum over all possible transitions from n_- to n_+.
- Weight each individual transition with e^{-S_0}.

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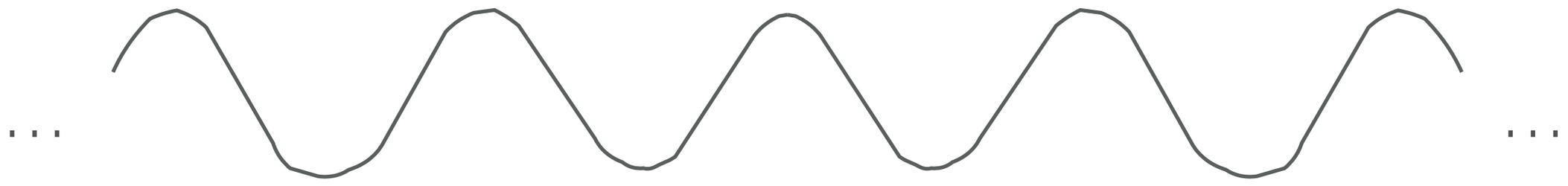

$$\langle n_+|e^{-HT}|n_-\rangle = \left(\frac{\omega}{\pi}\right)^{1/2} e^{-\frac{\omega T}{2}} \sum_{N=0}^{\infty}\sum_{\bar{N}=0}^{\infty} \frac{1}{N!\bar{N}!}(Ke^{-S_0}T)^{N+\bar{N}}\delta_{(N-\bar{N})-(n_+-n_-)}$$

Can distribute instantons and anti-instantons freely in time.

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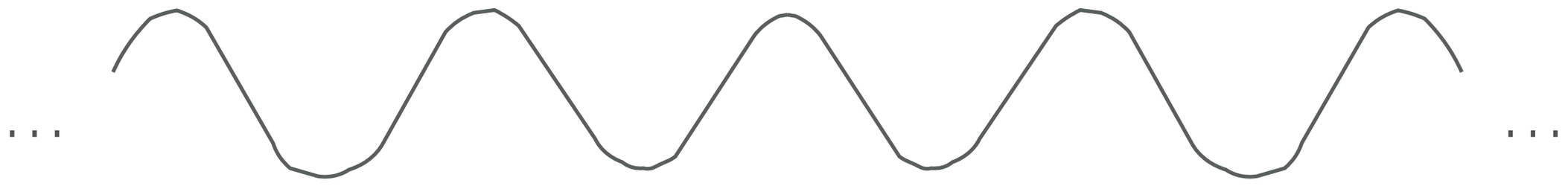


$$\langle n_+|e^{-HT}|n_-\rangle = \left(\frac{\omega}{\pi}\right)^{1/2} e^{-\frac{\omega T}{2}} \int_0^{2\pi} \frac{d\theta}{2\pi} e^{i(n_- - n_+)\theta} \exp\left(2KT\cos\theta e^{-S_0}\right)$$

We observe that the system has an eigenbasis $|\theta\rangle = \sum_n e^{in\theta}|n\rangle$.

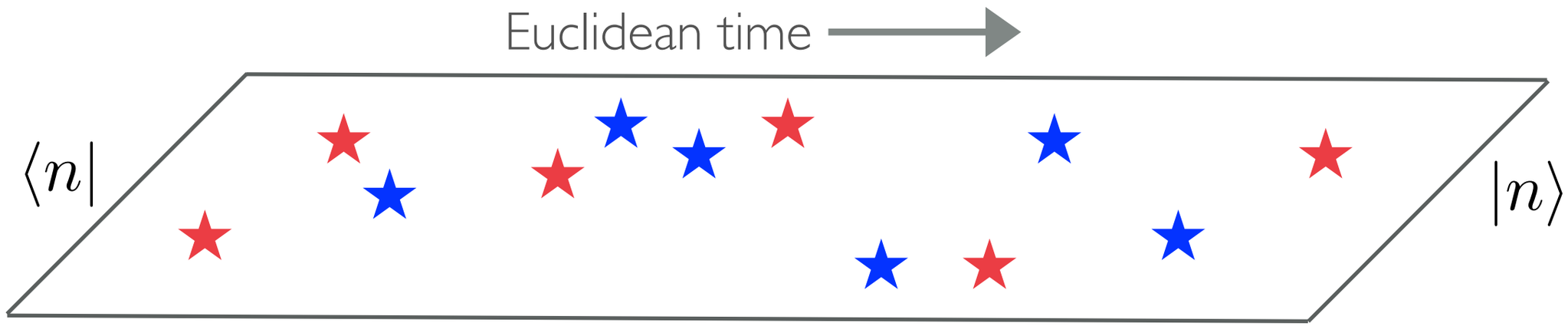
Read off the potential from the energy eigenvalues:

$$V = 2Ke^{-S_0}\cos\theta$$

Instanton Potentials

Field theory generalisation: e.g. Yang-Mills instantons

- Have Euclidean solutions associated with winding number n.
- Again, instanton: $n \to n+1$, anti-instanton: $n \to n-1$.
- **Crucial point:** Can distribute instantons and anti-instantons freely in spacetime:



- Calculation goes through as before: $V \sim e^{-S_0} \cos\theta$

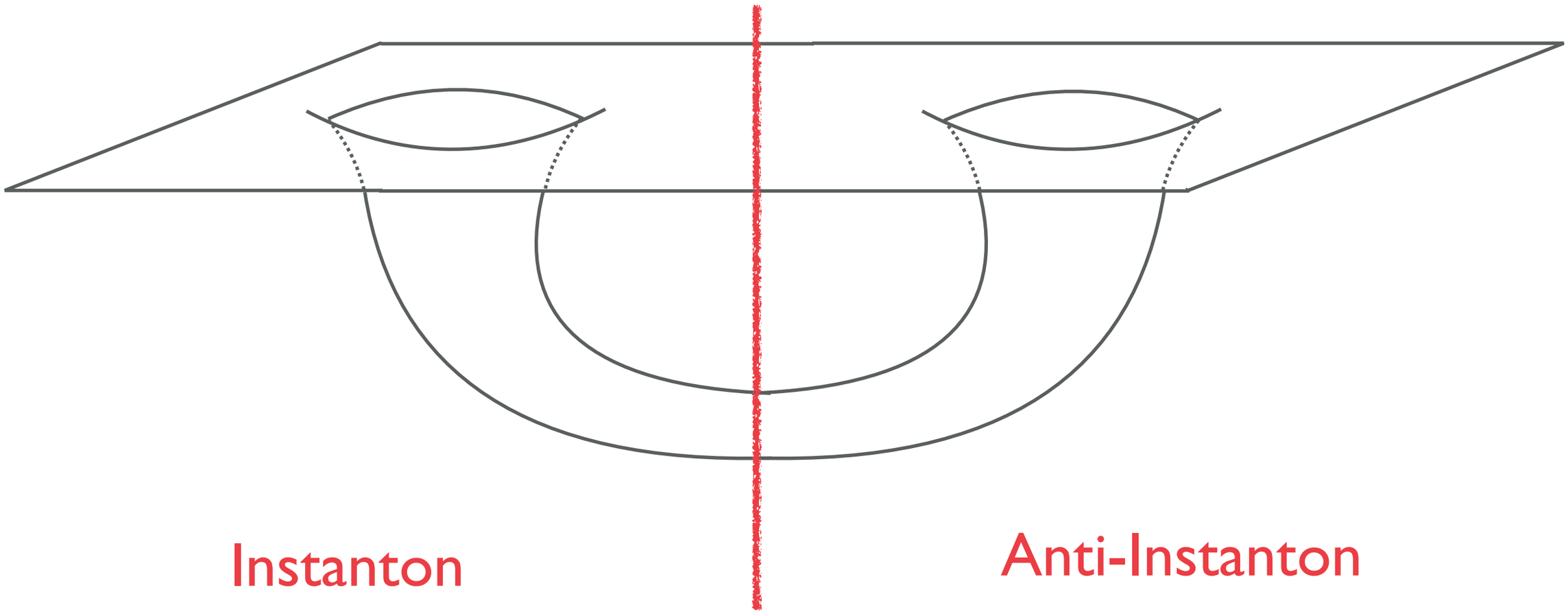
Instanton Potentials

Do Euclidean wormholes give rise to a potential?

- Gravitational Instantons exist in the presence of non-vanishing H_3 -flux.
- An instanton then corresponds to a transition from n units of flux to $n+1$. Similarly, for an anti-instanton $n \to n-1$.
- Apply Coleman's calculation, however…

Instanton Potentials

… wormholes always come as instanton/anti-instanton-pairs. Thus the inclusion of a wormhole leaves n unchanged overall.



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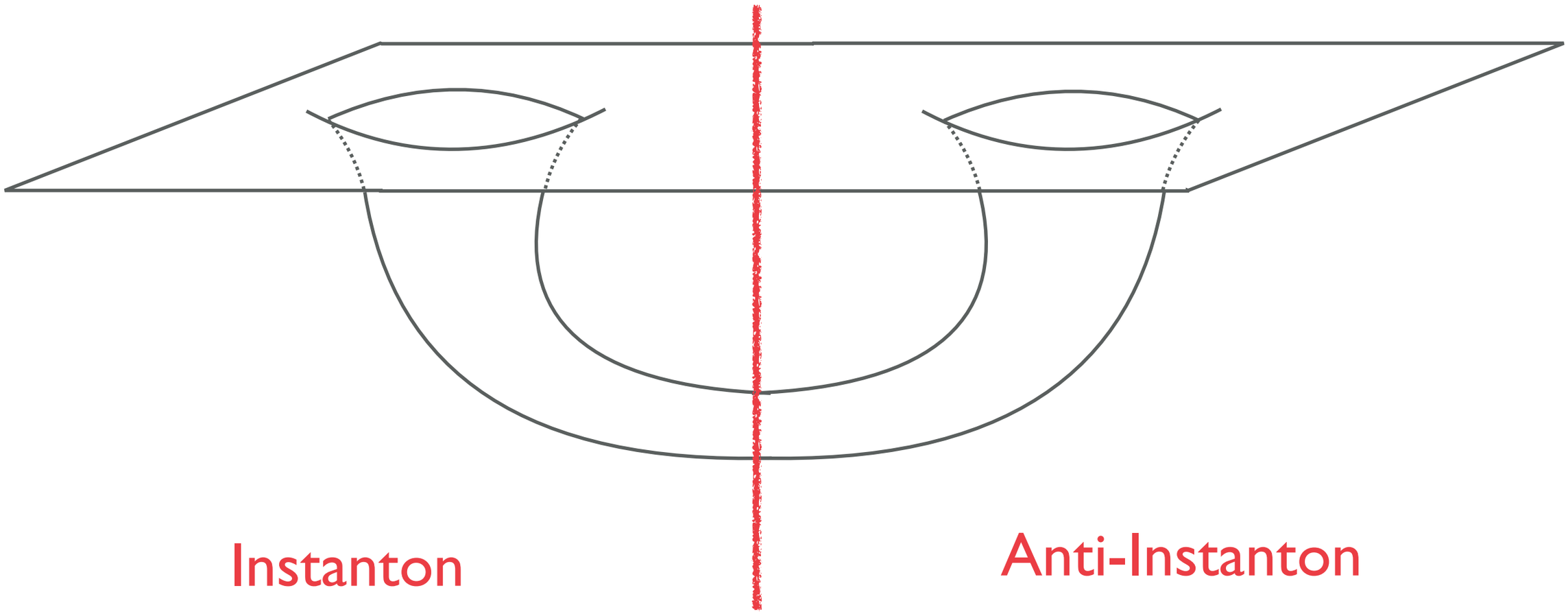
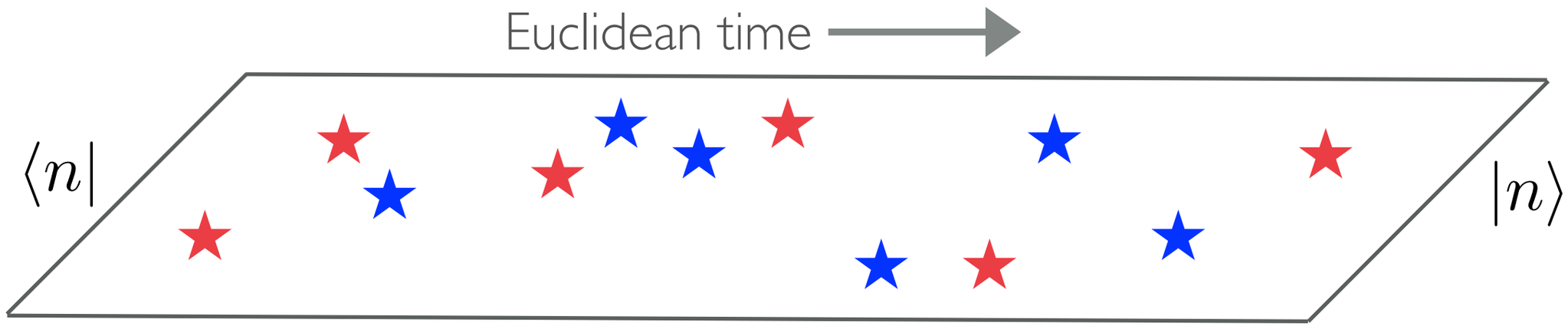


Crucial observation: we are still free to distribute the wormhole ends freely in spacetime.

Instanton Potentials

Euclidean Wormholes:

- Have n units of H_3 -flux.
- Again, instanton: $n \to n+1$, anti-instanton: $n \to n-1$.
- **Crucial point:** Can still distribute instantons and anti-instantons freely in spacetime:



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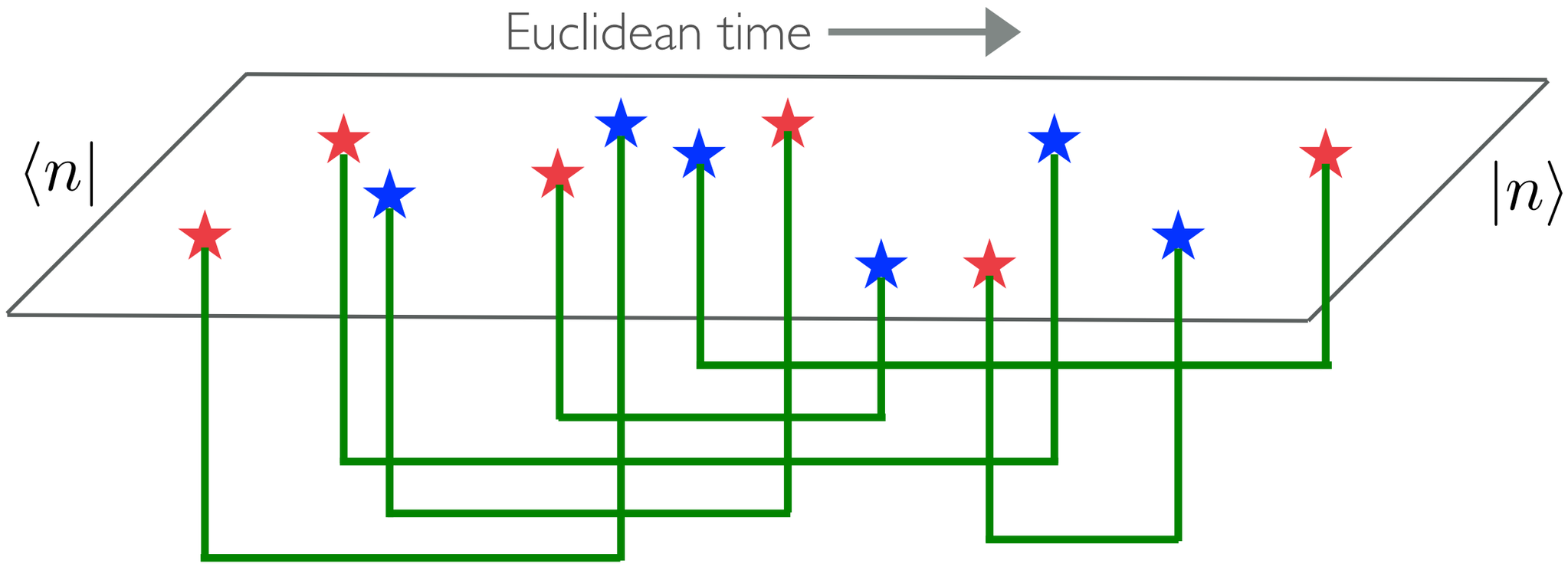

Result

Coleman's calculation applies:

- Again, have a set of eigenstates $|\theta\rangle = \sum_n e^{in\theta}|n\rangle$.
- Find a transition amplitude:

$$\langle\theta|e^{-HT}|\theta\rangle \sim \exp\left(2K\mathcal{V}Te^{-S_0}\cos\theta\right)$$

- Identify θ with the axion. Obtain

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- Expect this to generalize to:

$$V \sim \sum_n e^{-nS_0}\cos(n\theta)$$

Summary

Gravitational Instantons

- Model-independent constraints for large field axion inflation?

Euclidean Wormhole

- Wormholes always occur as instanton/anti-instanton pairs.
- Still, expect wormholes to give rise to an axion potential:

$$V \sim \sum_n e^{-nS_0} \cos(n\theta)$$

Open questions:

- What role do gravitational instantons and in particular wormholes play for the Weak Gravity Conjecture?

Constraining Inflation?

Wormhole case:

Potential $\delta V \sim \Sigma_n \; e^{-\frac{n}{f}} \cos(\frac{n\theta}{f})$

Scalar Curvature at throat $R \sim \frac{f}{n}$

Unsuppressed potential = curvature approaches Planck scale

Trust solution only up to a cutoff $R < \Lambda^2$. $\Rightarrow$ $\delta V \sim e^{-1/\Lambda^2}$

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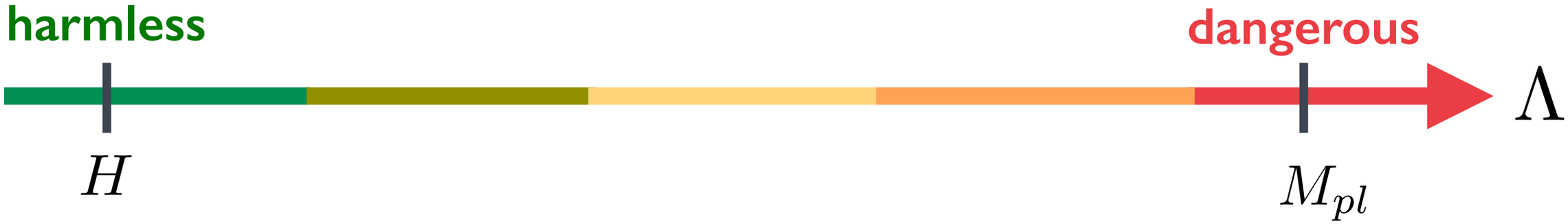
[Hebecker, Mangat, Rompineve, LW 2015]



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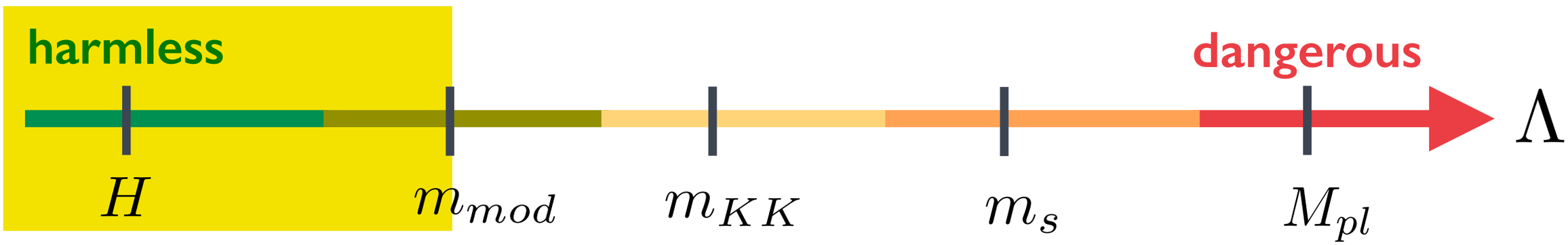

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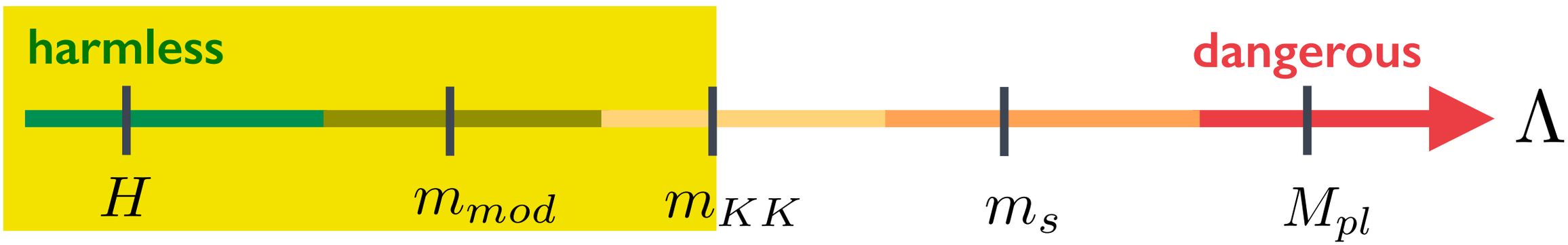

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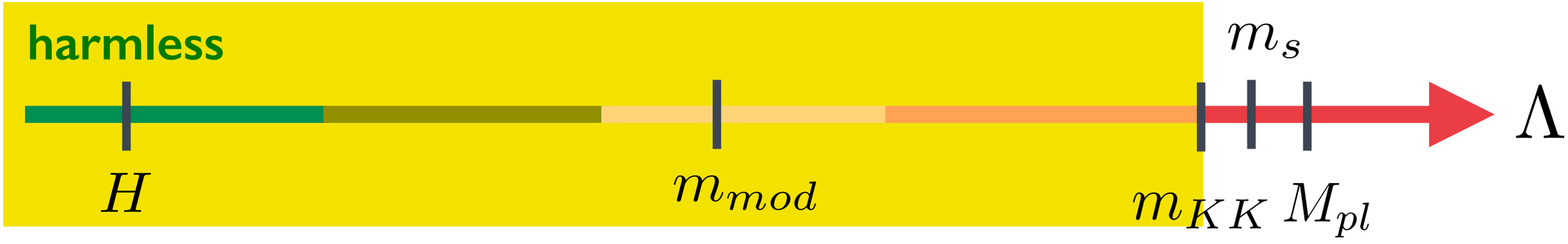

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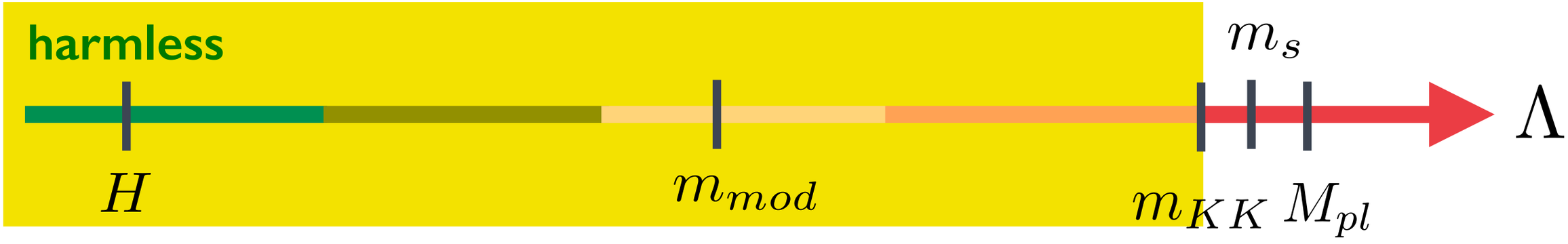
- In general, potential due to 4d gravitational instantons too small to rule out large field inflation.