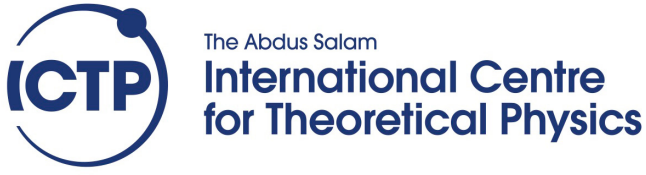
The vectorlike sector of F-theory on toric hypersurface fibrations

Damián K. Mayorga Peña

Based on **arXiv: 1607.xxxx**, in collaboration with
D. Klevers, P. Oehlmann, and J. Reuter



StringPheno, Ioannina, June 24, 2016

Plan

- Introduction and motivation
- Toric Hypersurfaces and conifold transitions in 6D
- Four dimensional models
- The vectorlike sector
- Conclusions and Prospects

Introduction and motivation

- F-theory has seen recent progress in the understanding of some global features with crucial implications for model building: U(1) symmetries, discrete symmetries, torsional symmetries...

  ...Park, Morrison, Grimm, Weigand, Braun, Keitel, Cvetic, Palti, Klevers, Borchmann, Piragua, Mayrhofer, Grassi, Song, Anderson, Garcia-Etxerbaría, Lin, Till, Oehlmann, Reuter...

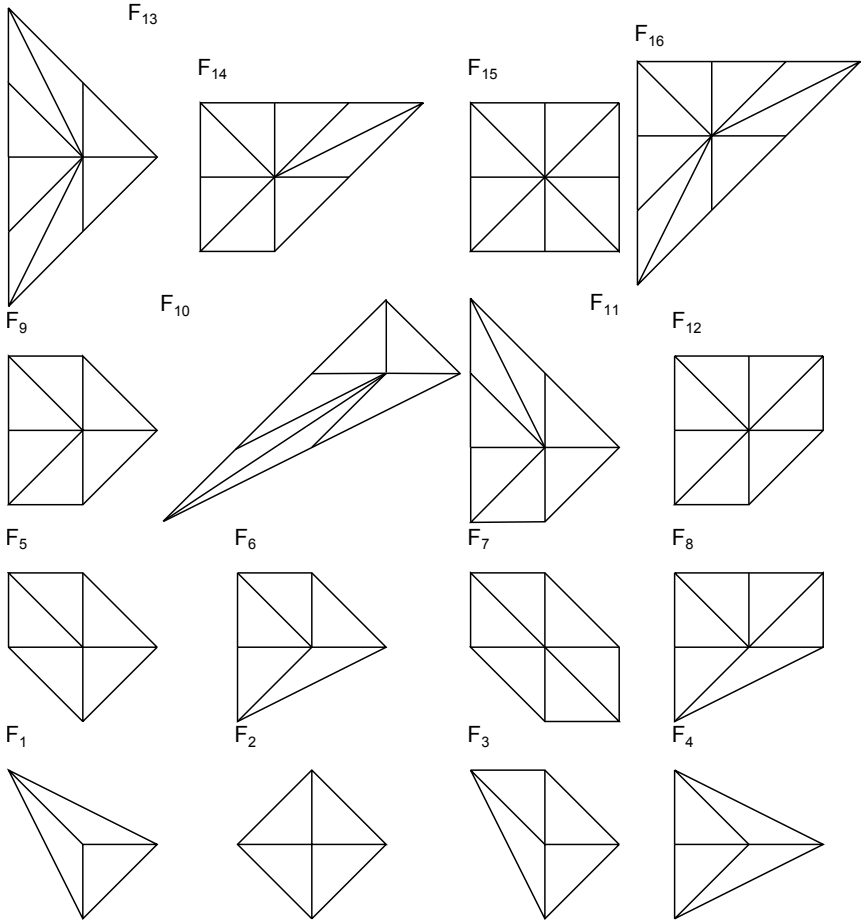
- Fiber presentations as hypersurfaces in 2D toric varieties have been at the core of these developments.
  $\rightarrow$ new opportunities for model building!

- Very simple models with appealing phenomenological features, plus a very rich structure:
  - all models seem connected by a network of extremal transitions.
  - intriguing duality relations between torsion and discrete symmetries.

  see P. Oehlmann's talk

- ...next step could be to look at their massless vectorlike spectra.

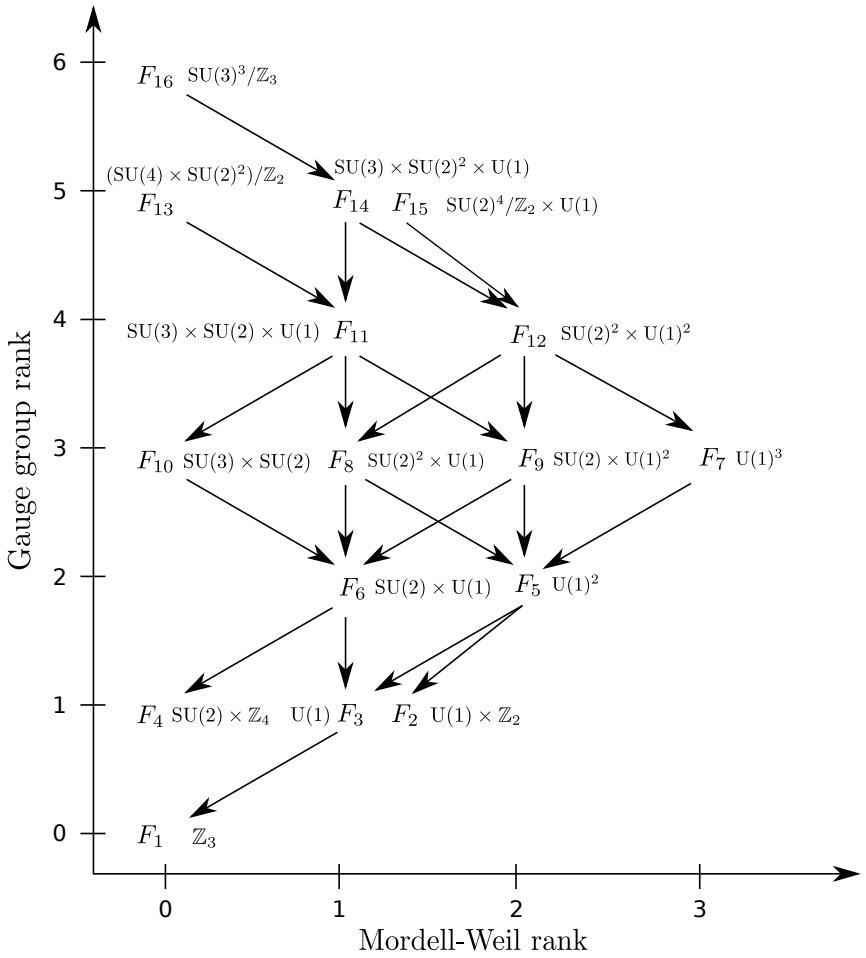
Toric hypersurfaces and conifold transitions in 6D



- Construct the fiber by cutting out a torus from any of the 16 2D toric ambient spaces.
- In this setup, many properties of the would be fibration can be read out from the fiber.
- Fiber the ambient space

$$\begin{array}{ccc} \mathbb{P}_{F_i} & \longrightarrow & \mathbb{P}^B_{F_i}(\mathcal{S}_7, \mathcal{S}_9) \\ & & \downarrow \\ & & B \end{array} \quad .$$

Toric hypersurfaces and conifold transitions in 6D

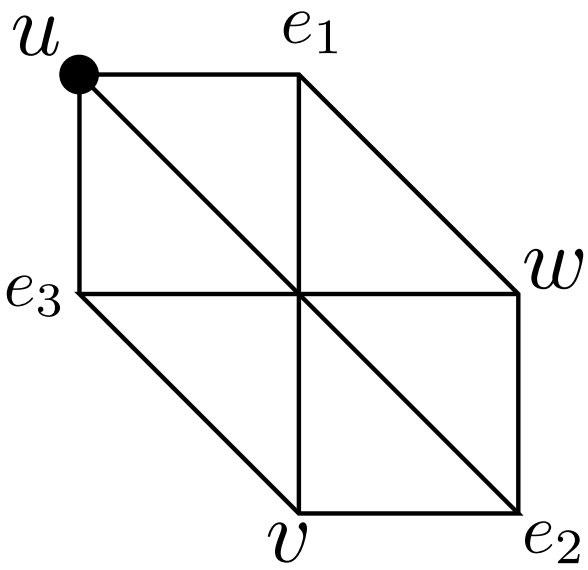


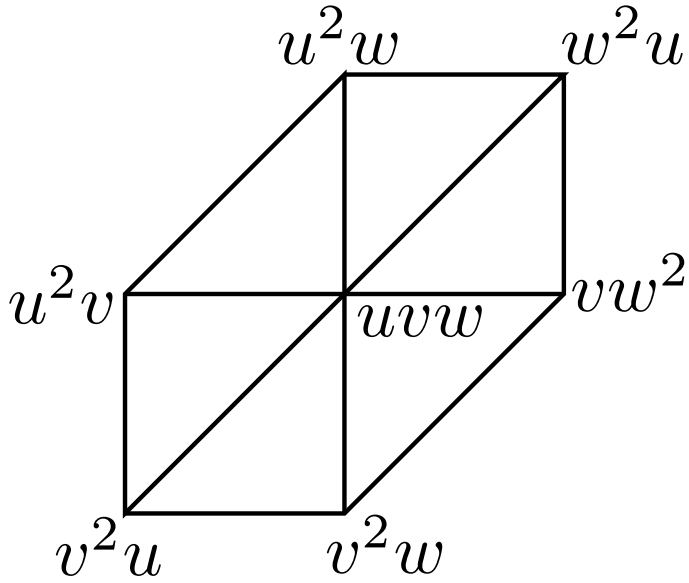
- Construct the fiber by cutting out a torus from any of the 16 2D toric ambient spaces.
- In this setup, many properties of the would be fibration can be read out from the fiber.
- and then cut out the fiber

$$\begin{array}{ccc} \mathcal{C}_{F_i} & \longrightarrow & X_{F_i} \\ & & \downarrow \\ & & B \end{array} \quad .$$

Toric hypersurfaces and conifold transitions in 6D

At the level of the polytope we see the possibility of a ”toric” Higgsing.





- This is suggestive of an extremal transition: *A resolution on the F_7 side and a deformation on the F_5 one.*

Toric hypersurfaces and conifold transitions in 6D

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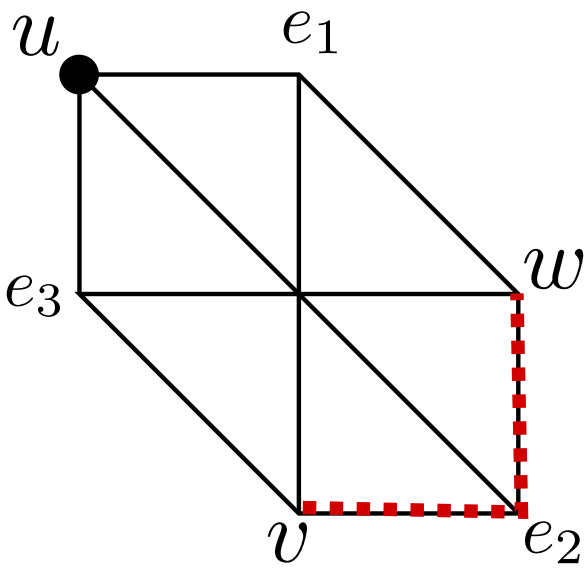

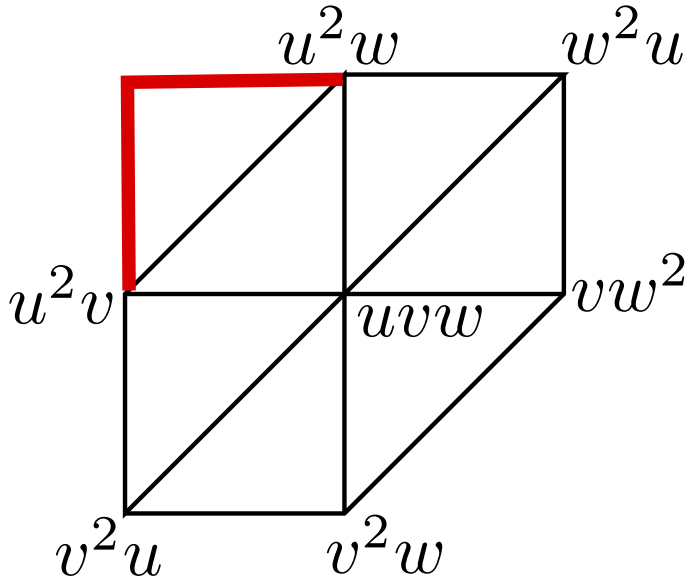

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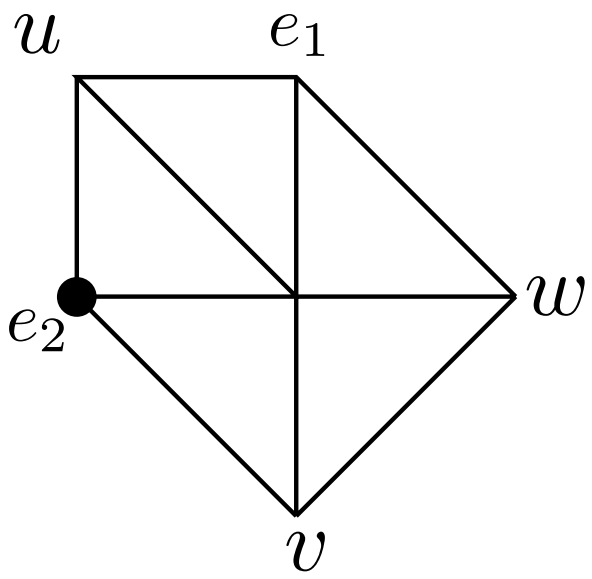
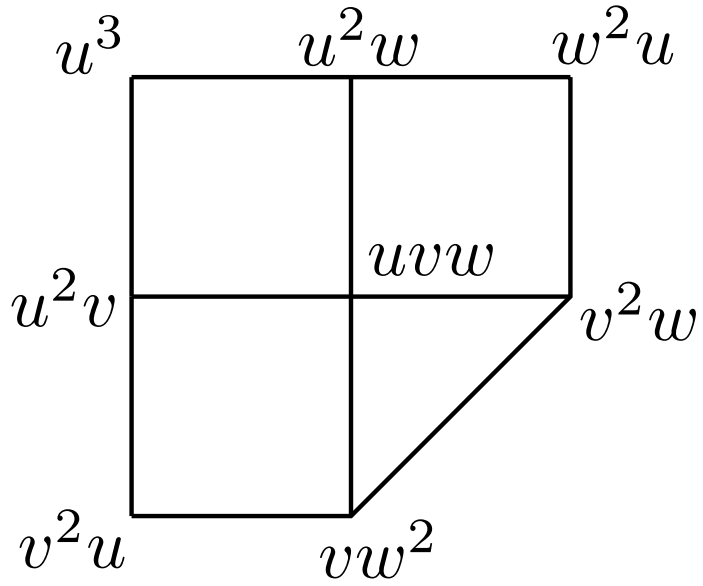
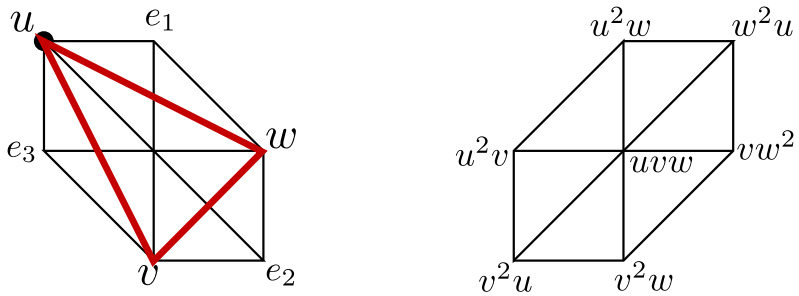




- This is suggestive of an extremal transition: *A resolution on the F_7 side and a deformation on the F_5 one.*

Toric hypersurfaces and conifold transitions in 6D

Consider the F_7 fiber on an "unresolved" $\mathbb{P}^2$.



$$p_{F7} = s_2 u^2 v + s_3 u v^2 + s_5 u^2 w + s_6 u v w + s_7 v^2 w + s_8 u w^2 + s_9 w^2 v = 0\,,$$

- p_{F7} is singular at $s_2 = s_5 = 0$ $[u : v : w] = [1 : 0 : 0]$. In the patch $u = 1$,

$$p_{F7} = s_2 v + s_5 w + \mathcal{O}(vw) = 0\,,$$

- in the very vicinity of $[u : v : w] = [1 : 0 : 0]$ one has

$$s_2 v + s_5 w = 0\,,$$

very reminiscent of the conifold equation.

Toric hypersurfaces and conifold transitions in 6D

- Considering the deformed conifold,

$$s_2 v + s_5 w + s_1 = 0\,,$$

one sees that this is consistent with the global fiber equation for p_{F_5}

$$p_{F5} = s_1 u^3 + s_2 u^2 v + s_3 u v^2 + s_5 u^2 w + s_6 u v w + s_7 v^2 w + s_8 u w^2 + s_9 w^2 v = 0\,,$$

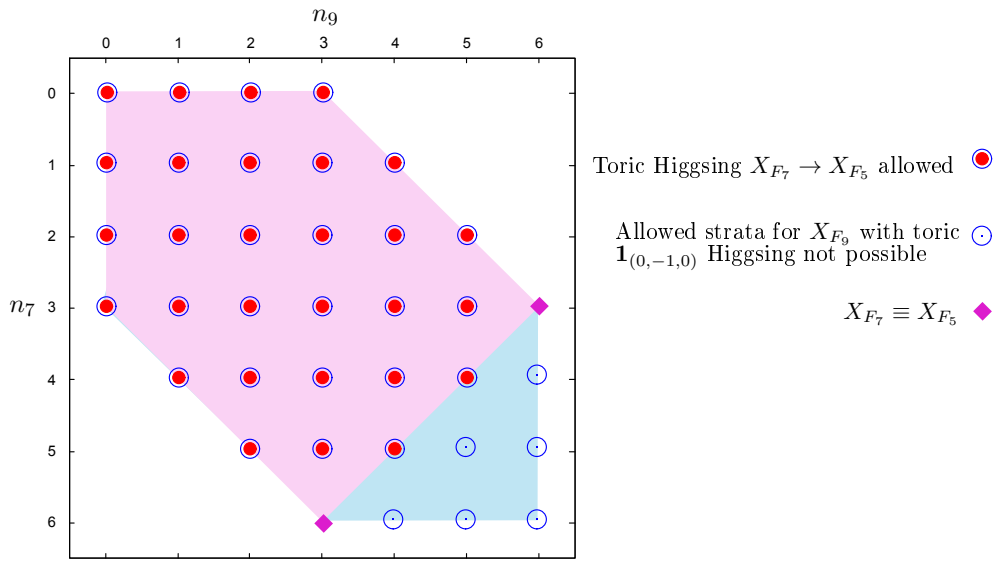
Toric hypersurfaces and conifold transitions in 6D

- On the field theory side one can show that hypermultiplet spectra before and after higgsing matches (for arbitrary base!).
- Moduli mismatch compensated by the neutral fields coming from the Higgs multiplet.

A concrete example with base $\mathbb{P}^2$

- Choosing the base fixes the allowed divisor classes of $\mathcal{S}_7$ and $\mathcal{S}_9$.

$$\mathcal{S}_7 = n_7 H_B\,, \qquad \mathcal{S}_9 = n_9 H_B\,, \qquad K_{\mathbb{P}^2} = -3H_B\,,$$



Four dimensional models

- Fix a complex three dimensional base $B = \mathbb{P}^3$.

$$\mathcal{S}_7 = n_7 H_B \,, \qquad \mathcal{S}_9 = n_9 H_B \,, \qquad K_B = -4 H_B \,,$$

- To construct the (chirality inducing) G_4 flux we have to work out $H_V^{(2,2)}(X_{F_i})$.
- G_4 is subjected to a quantization condition and must allow for a positive (integer) tadpole cancelling number of D3 branes

Witten'96, Sethi, Vafa, Witten'96, Gukov, Vafa, Witten'99

$$G_4 + \frac{c_2(X)}{2} \in H^4(X, \mathbb{Z}) \,, \qquad \frac{\chi(X)}{24} - \frac{1}{2}\int_X G_4 \wedge G_4 = n_{\mathrm{D3}} \in \mathbb{N} \,.$$

- There are also constraints on certain CS terms $\Theta_{AB} = \int_X G_4 \wedge D_A \wedge D_B$

Marsano, Schäfer-Nameki'11, Grimm, Hayashi'12, Cvetič, Grimm, Klevers'12

...we do not have an integral basis to check flux quantization. Instead we demand $\Theta_{AB} \in \mathbb{Z}/2$.

Intrilligator, Jockers, Mayr, Morrison, Plesser'12

- The chiralities: $\chi(\mathbf{R}) = n(\mathbf{R}) - n(\bar{\mathbf{R}}) = \int_{\mathcal{C}^w_{\mathbf{R}}} G_4$.

The vectorlike sector

- G_4 suffices for the computation of the chiralities, but further gauge data needed to deduce massless vectorlike pairs, in particular the M-th C_3 form.
  $\rightarrow$ Deligne cohomology and Chow grops.

Krause, Mayrhofer, Weigand'12; Bies, Mayrhofer,Pehle, Weigand'14

  $\rightarrow$ Cheeger Simons cohomology.

Intrilligator, Jockers, Mayr, Morrison, Plesser'12

- in agreement with the type IIB intuition, where massless fields get counted by cohomology groups

$$n(\mathbf{R}) = h^0(\mathcal{C}_\mathbf{R}, \mathcal{L}_\mathbf{R} \otimes \sqrt{K_{\mathcal{C}_\mathbf{R}}}), \quad n(\overline{\mathbf{R}}) = h^1(\mathcal{C}_\mathbf{R}, \mathcal{L}_\mathbf{R} \otimes \sqrt{K_{\mathcal{C}_\mathbf{R}}})$$

$$n(\mathbf{R}) - n(\overline{\mathbf{R}}) = \int_{\mathcal{C}_\mathbf{R}} c_1(\mathcal{L}_\mathbf{R})$$

  $\rightarrow$ one must relate the line bundle $\mathcal{L}_\mathbf{R}$ over $\mathcal{C}_\mathbf{R}$ to the expression for G_4 flux.

The vectorlike sector

Back to the transition $X_{F_7} \to X_{F_5}$

In X_{F_7}:

$$G_4 = a_3 H_B \hat{\sigma}(s_1) + a_4 H_B \hat{\sigma}(s_2) + a_5 H_B \hat{\sigma}(s_3) + a_6 (S_0^2 - (4 - n_7 n_9) H_B^2) \,,$$

Representation	Chirality
$\mathbf{1}_{(1,1,0)}$	$-(4+n_7-n_9)(-8+n_9)(a_3+a_4)$
$\mathbf{1}_{(0,-1,0)}$	$-(-8+n_7)(-8+n_9)a_4$
$\mathbf{1}_{(2,1,1)}$	$n_7(4+n_7-n_9)(2a_3+a_4+a_5-n_9a_6)$
$\mathbf{1}_{(0,1,1)}$	$(-8+n_7)(-4+n_7-n_9)(a_4+a_5)$
$\mathbf{1}_{(-2,-1,-2)}$	$n_7n_9(-2a_3-a_4-2a_5-(4-n_7-n_9)a_6)$
$\mathbf{1}_{(1,1,2)}$	$-(-4+n_7-n_9)n_9(a_3+a_4+2a_5-n_7a_6)$
$\mathbf{1}_{(1,0,0)}$	$(-2n_7^2-8(-8+n_9)+2n_7(4+n_9))a_3+n_7(-4+n_7-n_9)n_9a_6$
$\mathbf{1}_{(0,0,1)}$	$2(32+n_7(-4+n_9)+4n_9-n_9^2)a_5+n_7n_9(-4-n_7+n_9)a_6,$
$\mathbf{1}_{(1,0,1)}$	$(-2n_7(-4+n_9)+8(4+n_9))a_3+(32+8n_7+8n_9-2n_7n_9)a_5$ $+n_7n_9(-20+n_7+n_9)a_6$
$\mathbf{1}_{(1,1,1)}$	$-2(-32+n_7^2-4n_9+n_9^2-n_7(4+n_9))(a_3+a_4)$ $+(64+8n_7-2n_7^2+8n_9+2n_7n_9-2n_9^2)a_5$ $+(-20n_7n_9+n_7^2n_9+n_7n_9^2)a_6$

The vectorlike sector

Back to the transition $X_{F_7} \to X_{F_5}$

In X_{F_5}:

Cvetic, Grassi, Klevers, Piragua'13

$$G_4 = A_3 H_B \hat{\sigma}(s_1) + A_4 H_B \hat{\sigma}(s_2) + A_5 (S_0^2 - (4 - 4n_9 + n_7 n_9 - n_9^2) H_B^2)$$

Representation	Chirality
$\mathbf{1}_{(1,-1)}$	$-n_7(4 + n_7 - n_9)(A_3 - A_4)$
$\mathbf{1}_{(1,0)}$	$(2n_7^2 - (12 - n_9)(8 - n_9) - n_7(16 + n_9))A_3 + n_7 n_9(4 - n_7 + n_9)A_5$
$\mathbf{1}_{(-1,-2)}$	$n_9(4 - n_7 - n_9)(A_3 + 2A_4 + A_5(n_7 - 2n_9)$
$\mathbf{1}_{(-1,-1)}$	$(n_7^2 + n_7(n_9 - 20) + 2(12 - n_9)(4 + n_9))(A_3 + A_4)$ $-2n_9(4 - n_7 + n_9)(12 - n_9)A_5$
$\mathbf{1}_{(0,2)}$	$n_7 n_9(-2A_4 + (4 - n_7 + n_9)A_5)$
$\mathbf{1}_{(0,1)}$	$-(n_7(8 - n_7) + (12 - n_9)(4 - n_9))A_4 + n_9(4 - n_7 + n_9)(12 - n_9)A_5$

The vectorlike sector

- extra base section s_1 provides extra complex structure moduli, in addition to those in X_{F_7}

$$\chi(X_{F_5}) = \chi(X_{F_7}) + \Delta\chi$$

$$\Delta\chi = 6(\Delta h^{1,1} - \Delta h^{2,1} + \Delta h^{3,1}), \quad \Delta\chi = 6(-1 + \Delta h^{3,1})$$

but $\Delta\chi = 3\chi(\mathcal{C}_{\mathbf{R}}) = -3(2 - 2g_{\mathcal{C}_{\mathbf{R}}})$, $\Delta h^{3,1} = g_{\mathcal{C}_{\mathbf{R}}}$

- from the D3 tadpole cancelation (say, for $G_4^V = 0$), one sees that

$$n_{D3} = \frac{\chi(X_{F_7})}{24} = \frac{\chi(X_{F_5})}{24} + \left(\frac{\Delta\chi}{24} - \frac{1}{2}\int G_4' \wedge G_4'\right)$$

Intrilligator, Jockers, Mayr, Morrison, Plesser, Mayrhofer, Palti, Lin, Weigand, Till,...

The vectorlike sector

- Fluxes of the form G_4' induce potentials for some of the new $g_{\mathcal{C}_{\mathbf{R}}}$ moduli fields.
- this is consistent with a factorization

$$s_1 = s_1^+ s_1^-$$

with s_1^+, s_1^-, sections of $\mathcal{L}_{\mathbf{R}} \otimes \sqrt{K_{\mathcal{C}_{\mathbf{R}}}}$ and $\mathcal{L}_{\mathbf{R}}^* \otimes \sqrt{K_{\mathcal{C}_{\mathbf{R}}}}$.

- unrestricted moduli

$$h^0(\mathcal{C}_{\mathbf{R}}, \mathcal{L}_{\mathbf{R}} \otimes \sqrt{K_{\mathcal{C}_{\mathbf{R}}}}) + h^1(\mathcal{C}_{\mathbf{R}}, \mathcal{L}_{\mathbf{R}} \otimes \sqrt{K_{\mathcal{C}_{\mathbf{R}}}}) - 1 < g_{\mathcal{C}_{\mathbf{R}}}$$

- in a supersymmetric Higgsing we expect

$$n(\mathbf{R}) + n(\overline{\mathbf{R}}) - 1$$

neutral singlets

The vectorlike sector

The simplest case: No flux in X_{F_7}

$n_7 \backslash n_9$	0	1	2	3	4	5	6	7	8
0	$\begin{pmatrix} 84 \\ 385 \end{pmatrix}$	$\begin{pmatrix} - \\ 309 \end{pmatrix}$	$\begin{pmatrix} 56 \\ 241 \end{pmatrix}$	$\begin{pmatrix} - \\ 181 \end{pmatrix}$	$\begin{pmatrix} 34 \\ 129 \end{pmatrix}$	x	x	x	x
1	$\begin{pmatrix} - \\ 309 \end{pmatrix}$	$\begin{pmatrix} 56 \\ 246 \end{pmatrix}$	$\begin{pmatrix} - \\ 190 \end{pmatrix}$	$\begin{pmatrix} 35 \\ 141 \end{pmatrix}$	$\begin{pmatrix} - \\ 99 \end{pmatrix}$	$\begin{pmatrix} 19 \\ 64 \end{pmatrix}$	x	x	x
2	$\begin{pmatrix} 56 \\ 241 \end{pmatrix}$	$\begin{pmatrix} - \\ 190 \end{pmatrix}$	$\begin{pmatrix} 35 \\ 145 \end{pmatrix}$	$\begin{pmatrix} - \\ 106 \end{pmatrix}$	$\begin{pmatrix} 20 \\ 73 \end{pmatrix}$	$\begin{pmatrix} - \\ 46 \end{pmatrix}$	$\begin{pmatrix} 9 \\ 25 \end{pmatrix}$	x	x
3	$\begin{pmatrix} - \\ 181 \end{pmatrix}$	$\begin{pmatrix} 35 \\ 141 \end{pmatrix}$	$\begin{pmatrix} - \\ 106 \end{pmatrix}$	$\begin{pmatrix} 20 \\ 76 \end{pmatrix}$	$\begin{pmatrix} - \\ 51 \end{pmatrix}$	$\begin{pmatrix} 10 \\ 31 \end{pmatrix}$	$\begin{pmatrix} - \\ 16 \end{pmatrix}$	$\begin{pmatrix} 3 \\ 6 \end{pmatrix}$	x
4	$\begin{pmatrix} 34 \\ 129 \end{pmatrix}$	$\begin{pmatrix} - \\ 99 \end{pmatrix}$	$\begin{pmatrix} 20 \\ 73 \end{pmatrix}$	$\begin{pmatrix} - \\ 51 \end{pmatrix}$	$\begin{pmatrix} 10 \\ 33 \end{pmatrix}$	$\begin{pmatrix} - \\ 19 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 9 \end{pmatrix}$	$\begin{pmatrix} - \\ 3 \end{pmatrix}$	$\begin{pmatrix} - \\ - \end{pmatrix}$
5	x	$\begin{pmatrix} 19 \\ 64 \end{pmatrix}$	$\begin{pmatrix} - \\ 46 \end{pmatrix}$	$\begin{pmatrix} 10 \\ 31 \end{pmatrix}$	$\begin{pmatrix} - \\ 19 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 10 \end{pmatrix}$	$\begin{pmatrix} - \\ 4 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} - \\ - \end{pmatrix}$
6	x	x	$\begin{pmatrix} 9 \\ 25 \end{pmatrix}$	$\begin{pmatrix} - \\ 16 \end{pmatrix}$	$\begin{pmatrix} 4 \\ 9 \end{pmatrix}$	$\begin{pmatrix} - \\ 4 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} - \\ 0 \end{pmatrix}$	$\begin{pmatrix} - \\ - \end{pmatrix}$
7	x	x	x	$\begin{pmatrix} 3 \\ 6 \end{pmatrix}$	$\begin{pmatrix} - \\ 3 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} - \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} - \\ - \end{pmatrix}$
8	x	x	x	x	$\begin{pmatrix} - \\ - \end{pmatrix}$	$\begin{pmatrix} - \\ - \end{pmatrix}$	$\begin{pmatrix} 0 \\ - \end{pmatrix}$	$\begin{pmatrix} - \\ - \end{pmatrix}$	$\begin{pmatrix} - \\ - \end{pmatrix}$

The vectorlike sector

- Next one must find the line bundles $\mathcal{L}_{\mathbf{R}}$ over the matter curves.
  $\rightarrow$ For U(1) type fluxes $G_4 = a\sigma(S_i) \cdot H_B$, $c_1(\mathcal{L}_{\mathbf{R}}) = a\, q_{\mathbf{R}}\, H_B$
- What about the other type of fluxes $G_4 = b(S_0^2 + (\ldots)H_B^2)$?
  $\rightarrow$ Use the ansatz $c_1(\mathcal{L}_{\mathbf{R}}) = b\, s_{\mathbf{R}}\, H_B$, with the coeff. $s_{\mathbf{R}}$ analogous to the U(1) charge.

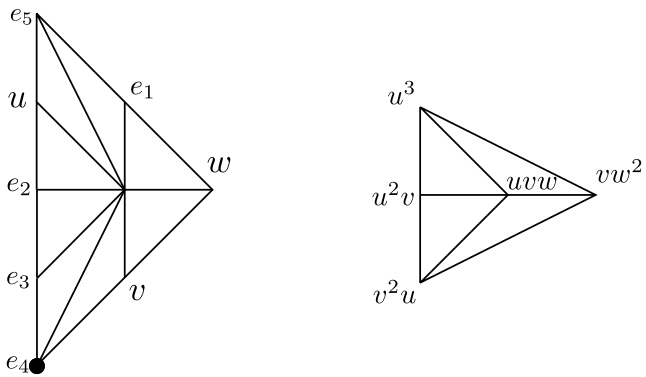
$$\chi(\mathbf{R}) = \int_{\mathcal{C}_{\mathbf{R}}} c_1(\mathcal{L}_{\mathbf{R}}) = \int_B c_1(\mathcal{L}_{\mathbf{R}}) D_a D_b$$

  from this we can deduce $s_{\mathbf{R}}$.

- then we can proceed with computation of cohomology groups with CohomCalg.

Blumenhagen, Jurke, Rahn, Roschy'10

The vectorlike sector



see M. Cvetic's talk

- **Gauge group:** $\mathrm{SU}(4) \times \mathrm{SU}(2)^2/\mathbb{Z}_2$
- **Flux:**

$$G_4 = -a_8 \left[S_0^2 + H_B \cdot \left(4S_0 + (12n_9 - n_7 n_9 - n_9^2) H_B - \tfrac{1}{2} n_9 D_1^{\mathrm{SU}(2)_1} \right.\right.$$
$$\left.\left. + \tfrac{1}{4}(n_7 + n_9 - 12)(3D_1^{\mathrm{SU}(4)} + 2D_2^{\mathrm{SU}(4)} + D_3^{\mathrm{SU}(4)}) \right) \right] .$$

Full Massless spectrum for $n_7 = n_9 = 4$, $a_8 = 2$

Representation	Locus	$s_{\mathbf{R}}$	$n(\mathbf{R})$	$n(\overline{\mathbf{R}})$	$\chi(\mathbf{R})$
$(\mathbf{1},\mathbf{2},\mathbf{2})$	$s_1 = s_3 = 0$	0	10	10	0
$(\mathbf{4},\mathbf{2},\mathbf{1})$	$s_1 = s_9 = 0$	$-[s3] \cdot H_B^2/4 = -1$	4	20	-16
$(\mathbf{4},\mathbf{1},\mathbf{2})$	$s_3 = s_9 = 0$	$[s1] \cot H_B^2/4 = 1$	20	4	16
$(\mathbf{6},\mathbf{1},\mathbf{1})$	$s_6 = s_9 = 0$	0	10	10	0

Conclusions and prospects

- ...steps towards obtaining the vectorlike spectrum in toric hypersurface fibrations...
- Fully fledged understanding of the Higgsing network...
  - extensions to the case of non complete intersections?
  - transitions with non-toric matter?
- ..look back at three family MSSM-like models with three families in order to identify the Higgs sector, vectorlike triplets etc...

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$\varepsilon\upsilon\chi\alpha\rho\iota\sigma\tau\varepsilon\upsilon!$